Specific Versus General Purpose Assets Under Uncertainty

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November 20, 2025

Abstract

Firms face a fundamental trade-off between flexibility and specialization when investing in new employees, equipment and technology. I develop a model in which production inputs differ in the specificity of their use: specific inputs can produce only a single good, while general inputs can produce one of many different goods. General inputs give the firm an option to reallocate assets in the future. The model shows how the expected time of exercise and price of risk attached to this reallocation payoff jointly determine the optimal investment composition. Contrary to conventional wisdom, the model predicts that firms operating in more volatile environments (1) invest more overall and (2) invest more in specific assets. I test and confirm these predictions for human, physical, and knowledge capital in a sample of U.S. firms. Using novel measures of skill specificity derived from job postings, I further show that the relationship between risk and investment specificity varies systematically with proxies for the cost of reallocating workers across geographies and business segments, consistent with the model.

Keywords: Uncertainty, Investment, Real Options, Asset Specificity

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I am deeply grateful to Bernard Dumas (Chair), Joël Peress and Massimo Massa for their continued support on this project. I would also like to thank Frederico Belo, Olivier Dessaint, Pierre Mabille, Naveen Gondhi, Martin Aragoneses, Sergei Glebkin, Boris Vallee, Ben Charoenwong, Stavros Panageas, Pat Akey, Xinwei Li and Antoine Hubert de Fraisse for helpful comments.

1 Introduction

In a survey of CEOs, 51% of respondents find that the potential to redeploy nonfinancial assets has a "large" or "a very large extent" impact on the firm's M&A decisions (Capron et al. 1998). This is supported by recent evidence from Tate and Yang 2024, who document internal labor reallocation after horizontal acquisitions as a source of value creation. General, redeployable assets give the firm flexibility. They include workers such as software engineers who can build just about anything, or equipment such as microscopes that can used from cancer research to studying respiratory illnesses.[1] The corporation can be viewed as an accumulation of production inputs that differ in the specificity of their use. Yet, the literature has mostly focused on the total quantity of investment rather than its individual composition. To the best of my knowledge, no paper has yet studied the joint investment decision when some inputs are specific and some inputs are general.

Understanding this joint investment decision is especially important during periods of high uncertainty. During these periods−periods that can last for several years−firms may favor general assets to more easily react to changes in the marketplace. Such flexibility creates value. On the other hand, expanding the firm's footprint in one market may require investment in potentially hard-to-reverse assets that have no other use but to help produce a single good. The multiproduct firm faces a natural trade-off between the benefits of investing in specific assets against the need for flexibility in the future (Teece 1982; Williamson 1985). This trade-off begs several questions. When is having flexible inputs most valuable? What discount rate should we use to evaluate a project if the assets are redeployable? How does it change firm value?

In this paper, I study the decision to invest in specific and general assets both theoretically and empirically. In the first part of the paper (Section 2), I build a dynamic model of the firm in which production inputs differ in the specificity of their use. The firm can use a combination of general and specific assets in each product line to produce a good. While general assets are production inputs that can be used to produce one of many different goods, specific assets are inputs that can be used to produce only one good. General assets give the firm an option to reallocate.

The key tension is an intertemporal trade-off between specific investments today versus a loss of flexibility in the future. The argument goes as follows. If general and specific assets are complementary in production, new investments in specific assets make the general assets in that business marginally more productive. However, when the firm redeploys the general

[1] As one concrete example, Apple Inc abandoned its 10-year program, Project Titan, to develop self-driving electric vehicles following a crash in the market for EVs in 2024. Instead of firing workers, Apple redeployed engineers to a new division centered on generative AI. See:
https://www.bloomberg.com/news/articles/2024-02-27/apple-cancels-work-on-electric-car-shifts-team-to-generative-ai?embedded-checkout=true

assets later on, it must forfeit the complementarities that were originally created alongside these specific investments. An unintended counsequence, by increasing the complementarities that are available, specific investments reduce the marginal benefit of using general assets somewhere else. As a result, the build-up of complementarities hinders the future movement of general assets. In other words, not only are specific investments specific to one use, they reduce flexibility later on.[2] Anticipating this friction, the firm avoids investing in projects that are too specific until it is clear that the intertemporal trade-off of losing flexibility becomes worthwhile.

This leads to an important result on the role of volatility. For a firm whose businesses are inherently volatile, flexibility is not as valuable. Intuitively, a manager in this firm is unlikely to retrench assets away from a product line at the first sign of trouble. Rather, precisely because it is volatile, he or she will count on the chance that the business can quickly bounce back. In fact, the manager may expect that it may take, say, 5-10 years to really find out if this business is a bust. But at these horizons, the present value of flexibility is relatively low due to time discounting, compounded by the firm's higher cost of capital.[3,4] As a result, avoiding or delaying investments in specific projects to preserve flexibility for the future is no longer worthwhile; the build-up of complementarities is not as punishing. In turn, volatile firms have higher appetites for specific investments. But with constant returns to scale, constant elasticity of substitution production technology, more specific investments imply more general investments as well. Total investment and specificity are increasing in firm-level risk.

I formalize this intuition using a continuous-time model of investment and reallocation. I study a two-product firm that uses a combination of general and specific assets in each product line to produce a good. The firm can take two actions: reallocate and invest. It

[2]One can think of investing in specific assets as changing the optimal exercise price and payoff of its reallocation option. If the firm is "near-the-money" to move assets out of this business, naturally it does not want to move the threshold further away and needlessly increase the chance this option is never exercised, and hence, worthless. Moreover, even if the option is exercised, the marginal benefits are lower. Ex-ante, to prevent its investment policy from killing the value of its reallocation option, the firm withholds investments in highly specific projects until this option is deep "out-of-money". On average, because the firm is often willing to invest in only a subset from the available menu of investment projects, with many projects over time, the firm is a generalist through its disproportionate investment in general-purpose projects.

[3]Reallocation inherits a property common to option theory: volatility expands the size of the no-action region. As stated, this is because just as quickly as one business can outperform the other, conditions can reverse. Since reallocation is costly, the no-action region expands to avoid inefficient round-trip transfers.

[4]The statement "the value of an option is increasing in the volatility of the underlying" does not apply to reallocation. Synthetically, it is a combination of a long call option on the business general assets are sent to (the destination) and a simultaneous short put on the business they're sent from (the sender). The 'dividends' are the difference in marginal productivity of general assets at the two businesses. A mean-preserving increase in the volatility of both businesses implies that higher expected payoffs from an increase in the likelihood of tail events at the destination business is offset by the simultaneous increase in the likelihood of tail events at the sender's, capping the upside. All that changes is the discount factor.

can either shift assets internally, moving general assets between its two businesses or it can acquire new assets externally, investing in projects in each business that differ in their degree of specificity. In addition, because reallocation is costly, the firm prefers to delay reallocation until the performance of the two product lines diverges sufficiently.[5] Using this new framework, I derive closed and semi-closed solutions for the firm's optimal policies using a new concept called double options: the value and optimal exercise of one option depends on the value and optimal exercise of another option and *vice-versa*.

The model generates two main predictions: (H1) investment is increasing in firm-level risk and (H2) specificity is increasing in firm-level risk. In addition, each prediction admits a conditional test based on reallocation costs and whether the volatilities of the product lines are the same or are different. The first conditional test (H1a) investigates the case in which the volatilities of a firm's product lines differ. In this setting, the option to reallocate also incorporates an option to change overall firm risk. The model predicts that the firm invests *less* overall in the more volatile division, but only if the reallocation cost is low.[6] The second conditional test (H2a) investigates the magnitude of reallocation costs, which can be interpreted as the cost of transforming general production inputs for alternative use. The model predicts that if reallocation is prohibitively costly, cross-sectional differences between specificity and volatility disappear because even less risky firms do not value flexibility.

In the second part of the paper (Sections 3 & 4), I empirically test the model using the cross-section of U.S. firms over the period 1976-2023.

First, I find empirical support for the prediction that investment is increasing in firm-level risk, and this result holds for physical, human, and knowledge capital. More specifically, when regressing investment rates on firm-level risk measured in a panel setting, I find positive and economically significant coefficients. Importantly, I measure firm-level risk over a 5-year window to reflect the long-horizon nature of option payoffs in my model. Quantitatively, I find that a one percentage point increase in volatility is associated with a 9.9%, 8.9%, and 10.6% increase in investment rates for each type of capital, comparable to magnitudes in the literature. Moreover, these results are robust to the choice of volatility, including equity, abnormal, or asset volatility, and is not due to a particular inclusion of controls or fixed effects specifications. Using the volatility of cash flows−which are only available at longer

[5] I assume adjustment costs are proportional to the quantity of general assets moved. In continuous time this implies the firm moves general assets not all the time, but in discontinuous, potentially lumpy, increments. More formally, the optimal control is a continuous process but not absolutely continuous. Hence, there is a region of no-action in which the firm waits. In addition, because reallocation is bi-directional, there are two push regions.

[6] Since volatility is reflected in each division's cost of capital, the option to reallocate assets from the more volatile to the less volatile business is an option to discount assets at a lower rate and hence, very valuable. The firm is willing to reallocate assets in this direction right away. As a result, the firm is especially hesitant to make specific investments in the riskier product line due to the potential to reallocate to the less risky one. Overall investment in the riskier division is lower, but this effect requires a low cost of reallocation.

horizons using quarterly data− yields consistently positive coefficients as well.

Second, I test the prediction that specificity is increasing in firm-level risk using new measures from job postings data, patents and the redeployability score from Kim and Kung 2016. In particular, motivated by the recent evidence of labor reallocation within firms (Tate and Yang 2015; Tate and Yang 2024), and labor reallocation as a motive for horizontal acquisitions (Capron et al. 1998), I focus on the specificity of human capital.

I construct new firm and industry measures of skill specificity using job vacancy postings from Lightcast (formerly Burning Glass) and interpret the firm's search and target hiring of generalist and specialist workers as a proxy for its investments in its stock of human capital.[7,8] This new measure of specificity is based on the average cosine similarity between a firm's vector of specialized skills against industries' outside of its own. Intuitively, it measures: how much overlap there is between the set of skills a firm consistently seeks to hire and the expertise required in other industries. For example, given that general-purpose skills such as SQL, Java, and Python are widely sought across all industries, the measure identifies which among Apple, Google, Microsoft, NVIDIA and Amazon sought relatively more generalist programmers as opposed to specialists in one product area.

In the cross-section, I show that firms with higher volatility exhibit significantly greater skill specificity: a one-percentage-point increase in volatility is associated with a 0.3 standard deviation increase in skill specificity.

To complement the labor-based measure, I construct additional measures of specificity based on the textual similarity of patents. Patents that exhibit little textual overlap with those in other industries reflect technologies with narrow applicability and high cost of adaptation for alternative use. I use this patent-based measure as a proxy for the specificity of knowledge and human capital required to produce such innovations. I find that in the cross-section, high volatility at the firm level is associated with higher specificity in comparable economics magnitudes to the job postings data.

As a final robustness check, I test the relation between specificity and uncertainty using

[7] The literature suggests that skilled labor is difficult to adjust. For recent evidence, see Dube et al. 2010, Dierynck et al. 2012, Ochoa 2013, Blatter et al. 2015, and Golden et al. 2020. Moreover, data from the Bureau of Labor Statistics Job Openings and Labor Turnover Survey (JOLTS) shows that average layoffs and voluntary quit rates across all major industry categories is quite low. Excluding services, layoffs & quits are roughly one-fifth that of average hiring rates, at around 1% of employment monthly. These magnitudes are comparable to physical capital depreciation rates and labor attrition rates used in the literature, for example Bloom 2009. Though the BLS does not track hiring & firing by occupation type, I also show that skilled labor, such as mathematicians and computer scientists, face much lower volatility in total employment compared to that of other professions (such as economists). Hence, I focus on the accumulation of skilled general and specific labor as a stock. Supplementary figures are in Appendix D.

[8] Bloom et al. 2021 show that the volume of job postings for an occupation category in Lightcast maps well to actual employment in that category using data from the Bureau of Labor Statistics. Moreover, I show that at the firm level, there is a correspondence between the volume of postings and next period employment for matched firms in Compustat.

physical capital. Following Kim and Kung 2016 who construct measures of asset redeployability from the Bureau of Economic Analysis (BEA) capital flow table, I study how multi-segment firm shifts their asset distribution across divisions that differ in their degree of redeployability via new investments and/or reallocation. In predictive regressions of redeployability on firm-level risk, I find that firms facing higher risk shift investment towards its *less* redeployable divisions. Quantitatively, a one-percentage-point increase in the firm's division-weighted volatility, adjusted for industry correlations, is associated with a one-third standard deviation decrease in redeployability.

Next, I return to labor data to test the second conditional prediction (H2a). Consistent with the model, I show that the positive relationship between specificity and firm-level risk is no longer pronounced when reallocation costs are low. In particular, I use the average pairwise distance between job positions as a proxy for the potential costs associated with reallocating workers; there are likely to be fewer impediments to transfer workers across differ units when say, everyone works in the same building. This directly increases the appeal of general-purpose projects. When the average distance is short or if the firm operates out of a single city, I find no relationship between specificity and firm-level risk.

Finally, I test the first conditional prediction (H1a) on the way multi-segment firms allocate investment across divisions with different levels of risk. Using a new measure of industry-pair relatedness as a proxy for the reallocation costs of transforming inputs for alternative use, I find that firms invest less in their riskier divisions. This underinvestment effect is strongest when the riskier division is more closely related to the firm's other segments. Conversely, I find no effect when the division is unrelated to the rest of the firm, emphasizing the role of reallocation costs. These findings are consistent with the model and demonstrates how the main predictions can be amplified or moderated as functions of key model parameters.

I contribute to three strands of literature: (1) the theory of capital, (2) specificity and irreversible investment, and (3) firm boundaries and multi-segment firms.

More recently in the theory of capital, Gârleanu et al. 2012 and Kogan et al. 2020 study embodied technical change within the context of asset pricing and the equity premium. Eisfeldt and Papanikolaou 2013 separate capital along physical and organizational capital, which is embodied in the firm's key talent, as sources of risk. Crouzet and Eberly 2023 separate capital along physical and intangible capital to explain the observed puzzle in lower aggregate investment despite rising corporate valuations. My paper contributes to this growing literature by separating capital inputs along the dimension of specificity.

I also build on the literature on irreversible investment and specificity. Williamson 1985 points out, "[d]o the prospective cost savings afforded by the special purpose technology justify the strategic hazards that arise as a consequence of their non-salvageable character?". Pindyck 1988 further motivates the literature on irreversible investment, "[i]rreversibility usually arises because capital is industry- or firm-specific, that is, it cannot be used in a different industry or by a different firm". I formalize two capital inputs, general and specific assets, that differ in how easily they can be reshaped inside the corporation. This is in contrast to Abel and Eberly 1996 who study separate cases when investment is either fully reversible, partially reversible or completely irreversible.

Empirically, Kermani and Ma 2023 comprehensively document the asset specificity of nonfinancial firms using liquidation values in bankruptcy filings. They find that physical assets are quite specific due to location specificity, depreciation and customization costs, though there is substantial variation by industry. In this paper, I provide labor and knowledge based measures of firm specificity using job postings data and patents.

In addition to the earlier papers cited, asset redeployment has been documented in specific industries as early as Miles 1982, Chandler 1990 and Anand and Singh 1997 in case study form. These findings broadly support the production-based approach in Belo 2010 and Cochrane 2020 who study producers' choice of state-contingent output. I contribute to this literature by using the firm's investments in general inputs as a proxy for its ability to smooth output across different states.

Finally, consider the literature on conglomerates. Most of the early works focused on the diversification discount and the other strand of this literature has focused on internal capital markets. Because of their focus, these papers use homogeneous capital inputs that are perfectly fungible across different segments. Moreover, differences in investment rates are often due to assumptions about decreasing returns to scale, for example Gomes and Livdan 2004. Recently, Dai et al. 2024 model the dynamic cash management problem of a diversified firm. Their work also characterizes the optimal investment policy when two segments differ in their level of risk, but they also assume homogeneous capital inputs. I show that when some inputs are specific to one business, the degree of relatedness and reallocation cost of transforming general assets matter just as much as the level of risk itself.

2 Theory. The multiproduct firm

Time is continuous. Suppose there are two product markets i and j, and consider a competitive firm that operates in both markets.

The firm generates operating profits in each period with a constant returns to scale and constant elasticity of substitution (CES) production technology,

$$\pi(t, A^i, A^j) = \left[\alpha(z_t^i)^\varepsilon + (1-\alpha)(k_t^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} A_t^i + \left[\alpha(z_t^j)^\varepsilon + (1-\alpha)(k_t^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} A_t^j \tag{1}$$

where A_t^i, A_t^j are stochastic processes that drive the prices of goods in each market,

$$\begin{aligned}\mathbf{d}A_t^i &= A_t^i \mu^i \mathbf{d}t + A_t^i \sigma^i \mathbf{d}B_t^i \\ \mathbf{d}A_t^j &= A_t^j \mu^j \mathbf{d}t + A_t^j \sigma^j \mathbf{d}B_t^j,\end{aligned}$$

and $\mathbf{d}B_t^i \mathbf{d}B_t^j / \mathbf{d}t = \rho$. I assume the parameters $\{\alpha, \varepsilon, \theta\}$ are the same in both product lines; α controls the relative share, ε relates to the elasticity of substitution between general and specific assets (complements at $\varepsilon = 0$, substitutes at $\varepsilon = 1$), and θ is the degree of returns to scale. Unless otherwise stated, $\theta = 1$ throughout this paper (constant returns to scale).

I write k_t^i to denote the amount of general capital allocated to the production of good i, and similarly, z_t^i to denote the installed specific, dedicated assets used in the production of this good. The inputs k_t^i, k_t^j are the fungible, multipurpose assets within a firm that create flexibility: the firm can use these assets to produce different goods, so they capture partial reversibility within the firm's boundaries. They encompass inputs in general, and though written with k, not necessarily restricted to physical capital. In contrast, z_t^i, z_t^j are specific and nontransferable in their use. While likely aggressive simplification, the assumption highlights the non-salvageability of dedicated and specialized production technology, and is meant to encapsulate all the investments a firm makes in a business that have no other purpose than to help produce a single good.

2.1 Firm Value with Reallocation and No Investment

I hold off the introduction of investment until Section 2.2. Because investment depends heavily on the way the specificity of a project affects flexibility in the future, I first focus on reallocation and its comparative statics.

Without any external investment, z_t^i and z_t^j lose their time subscripts and are held fixed at their initial values, z^i, z^j.

Suppose the firm is able to move general capital by reallocating Δk between k^i and k^j at an adjustment cost proportional to the quantity moved.[9] Examples of reallocation include re-purposing equipment to be used in a firm's more profitable segment or sending a worker to help develop a different product within the firm. General assets suffer zero loss during

[9]In continuous time, proportional costs imply that the firm moves small increments of general assets from one product line to the other only after the stochastic price ratio of the two goods reaches a critical threshold. Unlike with quadratic costs in which the firm can react to every minor change, proportional costs ensure that the firm decides to move assets only when the performances of the two products diverge past a defined point. The intuition is that with proportional costs, continuously adjusting would be suicidal due to the infinite first order variation of Brownian motion, as opposed to the finite quadratic variation with quadratic costs, a point made in Dumas and Luciano 2017. The optimal control therefore splits the state-space into a region of no-action in which the firm stays put, and a push region in which the firm reallocates. I make this assumption due to the observation that there is a limit to the frequency at which factories (and people) are repurposed in practice. Proportional costs provide the appropriate type of optimal control.

transit; the total stays at K across repeated movement. Instead, the adjustment cost can be interpreted as the cost of transforming these assets for alternative use. For physical capital, one can think of transport or customization costs. For human capital, the adjustment cost can include relocation expenses but is also more figurative; it takes a toll on the worker to change job functions.

Let $\Delta k_t^{i\to j}$ denote the quantity of general capital the firm moves from i to j at time t, and similarly for $\Delta k_t^{j\to i}$. I make the following assumption about the costs to reallocate.

Assumption 1. The cost to move $\Delta k_t^{i\to j}$ from i to j is $cA_t^i \times \Delta k_t^{i\to j}$ and the cost to move $\Delta k_t^{j\to i}$ from j to i is $cA_t^j \times \Delta k_t^{j\to i}$.

The cost structure says that when a product line is doing poorly, it is cheaper to move assets out of this market, and is proportional to the quantity moved. I include the level of the stochastic variable to preserve homogeneity of the firm's value function once a change of variable is applied.

The firm's problem can be written,

$$\max_{\{\Delta k_t^{i\to j}, \Delta k_t^{j\to i}\}} \mathbb{E}_0^{\mathbb{Q}}\left[\int_0^\infty e^{-rt}\left(\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k_t^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} \times A_t^i + \left[\alpha(z^j)^\varepsilon + (1-\alpha)(k_t^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} \times A_t^j\right)\mathbf{d}t\right]$$
$$-\mathbb{E}_0^{\mathbb{Q}}\left[\int_0^\infty e^{-rt}\left(cA_t^i \times \Delta k_t^{i\to j} + cA_t^j \times \Delta k_t^{j\to i}\right)\mathbf{d}t\right]$$

such that

$$\begin{aligned}
&k_t^i + k_t^j = K\\
&k_t^i \geq 0, k_t^j \geq 0\\
&k_t^i = k_{t^-}^i - \Delta k_t^{i\to j} + \Delta k_t^{j\to i}\\
&k_t^j = k_{t^-}^j - \Delta k_t^{j\to i} + \Delta k_t^{i\to j}
\end{aligned}$$

I assume markets are complete, Modigliani-Miller theorem holds, and there exists an exogenous stochastic discount factor in the economy, $\mathbf{d}M_t/M_t = -r\mathbf{d}t - \kappa\mathbf{d}B_t$.[10]

It is convenient to consider the change of variable $s_t = \log A_t^j - \log A_t^i$. This amounts to focusing on the ratio A_t^j/A_t^i. A large negative value of s_t implies we want to shift capital away from j into i while a large positive value means from i into j. Furthermore, for brevity, I rewrite $\Delta k_t^{i\to j}$ as $\mathbf{d}k$. The optimal decision rule is to transfer $\mathbf{d}k$ from i to j whenever s_t moves above a certain threshold $\bar{s}$, and from j to i whenever s_t moves below $\underline{s}$.

By the homogeneity of the value function, $A^i J\left(1, \log A^j/A^i, k^i, k^j\right) = A^i H(s, k^i, k^j)$, and

[10]If $\mathbf{d}B_t = \begin{bmatrix}\mathbf{d}B_t^i & \mathbf{d}B_t^j\end{bmatrix}^\top$, then κ is 1×2 and I write $\kappa^{(1,0)}$ to denote the first element.

the Hamilton-Jacobi-Bellman (HJB) equation is given by

$$
\begin{aligned}
\tilde{r}H(s,k^i,k^j) = \Big[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}} + \Big[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}} e^s \\
+ \tilde{\mu}\frac{\partial H(s,k^i,k^j)}{\partial s} + \frac{1}{2}(\tilde{\sigma})^2\frac{\partial^2 H(s,k^i,k^j)}{\partial s^2}
\end{aligned} \tag{2}
$$

with four boundary conditions

$$
\frac{\partial H(\bar{s},k^i,k^j)}{\partial k^j} - \frac{\partial H(\bar{s},k^i,k^j)}{\partial k^i} = c \quad ; \quad \frac{\partial H(\underline{s},k^i,k^j)}{\partial k^i} - \frac{\partial H(\underline{s},k^i,k^j)}{\partial k^j} = ce^{\underline{s}} \tag{3}
$$

$$
\frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^j \partial s} - \frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^i \partial s} = 0 \quad ; \quad \frac{\partial^2 H(\underline{s},k^i,k^j)}{\partial k^i \partial s} - \frac{\partial^2 H(\underline{s},k^i,k^j)}{\partial k^j \partial s} = ce^{\underline{s}} \tag{4}
$$

where

$$
\begin{aligned}
&\tilde{\mu} = (\mu^j - \kappa^{(0,1)}\sigma^j) - (\mu^i - \kappa^{(1,0)}\sigma^i) - \left(\frac{1}{2}(\sigma^i)^2 + \frac{1}{2}(\sigma^j)^2 - \rho\sigma^i\sigma^j\right) \\
&\tilde{\sigma}^2 = (\sigma^j)^2 + (\sigma^i)^2 - 2\rho\sigma^j\sigma^i \quad ; \quad \tilde{r} = r - u^i + \kappa^{(1,0)}\sigma^i
\end{aligned} \tag{5}
$$

The two boundary conditions in Equation (3) enforce continuity of the value function at the reflection points, $\bar{s}$ and $\underline{s}$, and the two conditions in Equation (4) are optimality conditions on the placement of the barriers. In particular, the first boundary condition states that at the reflection point $\bar{s}$, the marginal value of a unit of general capital at j minus the marginal value of a unit of general capital at i is equal to c.

Proposition 1 (Scaled Firm Value)**.** Define $\delta^i \doteq r - \mu^i + \kappa^{(1,0)}\sigma^i$, $\delta^j \doteq r - \mu^j + \kappa^{(0,1)}\sigma^j$ for product specific discount rates. Moreover, let

$$
\phi\pm = \frac{-\tilde{u} \pm \sqrt{2\tilde{r}\tilde{\sigma}^2 + \tilde{u}^2}}{\tilde{\sigma}^2}
$$

Scaled firm value is

$$
\begin{aligned}
H(s,k^i,k^j) = \frac{[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}}{\delta^i} + \frac{[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon]^{\frac{\theta}{\varepsilon}}}{\delta^j} e^s \\
+ \underbrace{\mathbb{C}_1(k^i,k^j)e^{\phi^+ s}}_{\substack{\text{option to transfer} \\ \text{from } i\to j}} + \underbrace{\mathbb{C}_2(k^i,k^j)e^{\phi^- s}}_{\substack{\text{option to transfer} \\ \text{from } j\to i}}
\end{aligned} \tag{6}
$$

where $\mathbb{C}_1(k^i, k^j)$ and $\mathbb{C}_2(k^i, k^j)$ are given by

$$\begin{aligned}
\mathbb{C}_1(k^i, k^j) = &- \frac{\left[[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon]^{\frac{\theta}{\varepsilon}} - [\alpha(z^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}\right]}{\delta^i} \times \Upsilon_{1,1} \\
&+ \frac{\left[[\alpha(z^j)^\varepsilon + (1-\alpha)(k^i + k^j)^\varepsilon]^{\frac{\theta}{\varepsilon}} - [\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon]^{\frac{\theta}{\varepsilon}}\right]}{\delta^j} \times \Upsilon_{1,2} - ck^i \times \Upsilon_{1,3} \quad (7) \\
\mathbb{C}_2(k^i, k^j) = &\frac{\left[[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i + k^j)^\varepsilon]^{\frac{\theta}{\varepsilon}} - [\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}\right]}{\delta^i} \times \Upsilon_{2,1} \\
&- \frac{\left[[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon]^{\frac{\theta}{\varepsilon}} - [\alpha(z^j)^\varepsilon]^{\frac{\theta}{\varepsilon}}\right]}{\delta^j} \times \Upsilon_{2,2} - ck^j \times \Upsilon_{2,3} \quad (8)
\end{aligned}$$

and the six constants $\Upsilon_{1,1}, \Upsilon_{1,2}, \Upsilon_{1,3}, \Upsilon_{2,1}, \Upsilon_{2,2}, \Upsilon_{2,3}$ are exponential functions of $\bar{s}, \underline{s}$ and provided in Appendix Equation A3.

Proof. See Appendix A.1. □

In Equation (6), the fact that ϕ^+ is a positive number and ϕ^- is a negative number permits an easy interpretation of the integration constants $\mathbb{C}_1(k^i, k^j)$ and $\mathbb{C}_2(k^i, k^j)$. When j is doing very well relative to i, s is a number greater than 0 (since it is the log ratio), so $\mathbb{C}_1(k^i, k^j)e^{\phi^+ s}$ naturally corresponds to the value of reallocating from i to j. Furthermore, this option becomes more valuable as s increases since it can be continuously exercised past $\bar{s}$ until the stock of k^i is exhausted. A similar argument holds for $\mathbb{C}_2(k^i, k^j)e^{\phi^- s}$.

For ease of interpretability and to directly see firm value as a function of the barriers, it is convenient to consider the case $\varepsilon \to 0, \alpha = 0.5, \theta = 2$, and consider single-direction reallocation when reallocation can occur irreversibly from $i \to j$. Then,

Corollary 1. Let $\varepsilon \to 0, \alpha = 0.5, \theta = 2$ and consider single direction transfer from $i \to j$. Then,

$$H(s, k^i, k^j) = \frac{z^i k^i}{\delta^i} + \frac{z^j k^j}{\delta^j} e^s + k^i \left[\frac{z^j}{\delta^j} e^{\bar{s}} - \frac{z^i}{\delta^i} - c\right] e^{-\phi^+(\bar{s} - s)} \quad (9)$$

and the optimal point of transfer $\bar{s}$ given by

$$\bar{s} = \log\left(\frac{\phi^+}{\phi^+ - 1} \frac{z^i/\delta^i + c}{z^j/\delta^j}\right) \quad (10)$$

which, when plugged into (9), leads to

$$H(s, k^i, k^j) = \frac{z^i k^i}{\delta^i} + \frac{z^j k^j}{\delta^j} e^s + \underbrace{k^i \left[\frac{(\phi^+ - 1)^{\phi^+ - 1}}{(\phi^+)^{\phi^+}} \left(\frac{z^j}{\delta^j}\right)^{\phi^+} \left(\frac{z^i}{\delta^i} + c\right)^{1 - \phi^+}\right]}_{\doteq \mathbb{C}_1} e^{\phi^+ s} \quad (11)$$

Proof. See Appendix A.2. □

Focusing on the interpretation of Equation (9), the expression $k^i[z^j/\delta^j e^{\bar{s}} - z^i/\delta^i - c]e^{-\phi^+(\bar{s}-s)}$ loosely says that a transfer of increment $\mathbf{d}k$ from i to j leads to a marginal loss of z^i/δ^i but a marginal gain of $z^j/\delta^j e^{\bar{s}}$. Since proportional costs imply that it is optimal to wait until the threshold is reached, $\bar{s}$ is chosen to maximize this trade-off. "Moneyness" is captured by the final term in the expression, $e^{-\phi^+(\bar{s}-s)}$. The individual term $e^{-\phi^+\bar{s}}$ is a penalty that can be thought of in the following way: any quantity that pushes up the optimal exercise boundary $\bar{s}$ prolongs the time it takes for s to reach $\bar{s}$. As a result, the potential benefits of reallocation must be discounted further since these cash flows will accrue at a later date.[11]

The equations for CES production in (7) and (8) are similar; each option value is the difference in marginal productivity of general assets at one business versus the other if the option is fully exercised.

The below corollary shows comparative statics for $\bar{s}$ and $\mathbb{C}_1$ with respect to parameters that are central to the rest of the paper.

Corollary 2. Let $\varepsilon \to 0, \alpha = 0.5, \theta = 2$ and consider single direction transfer from $i \to j$. Moreover, consider the range of parameter values such that $\tilde{\sigma} > \tilde{r}^{3/2}$.[12] Then,

$$\textbf{i.}\ \frac{\partial \bar{s}}{\partial z^i} > 0 \quad \textbf{ii.}\ \frac{\partial \bar{s}}{\partial c} > 0 \quad \textbf{iii.}\ \frac{\partial \bar{s}}{\partial \sigma^i} < 0 \quad \textbf{iv.}\ \frac{\partial \bar{s}}{\partial \sigma^j} > 0 \quad \textbf{v.}\ \frac{\partial \bar{s}}{\partial \sigma}\Big|_{\sigma^i=\sigma^j=\sigma} > 0 \tag{12}$$

$$\frac{\partial \mathbb{C}_1}{\partial z^i} < 0 \qquad \frac{\partial \mathbb{C}_1}{\partial c} < 0 \qquad \frac{\partial \mathbb{C}_1}{\partial \sigma^i} > 0 \qquad \frac{\partial \mathbb{C}_1}{\partial \sigma^j} < 0 \qquad \frac{\partial \mathbb{C}_1}{\partial \sigma}\Big|_{\sigma^i=\sigma^j=\sigma} < 0 \tag{13}$$

The comparative statics can be derived by either implicitly or explicitly differentiating $\bar{s}$ in (10) and the $\mathbb{C}_1$ term in (11) with respect to the parameters. They hold generally in the bi-directional case with CES production when values of the share parameter α are away from 0 and 1 and $\varepsilon < 1$.

i. First, $\partial\bar{s}/\partial z^i > 0$. Because general assets are not scale free, there is an opportunity cost of using $\mathbf{d}k$ in j instead of i. Provided that specific and general assets are not perfect substitutes in production ($\varepsilon < 1$), as the stock of specific production technology used to produce good i increases, the marginal value of using $\mathbf{d}k$ in product market j decreases. This can be seen as just subtracting a bigger number inside the brackets $[z^j/\delta^j e^{\bar{s}} - z^i/\delta^i - c]$ in Equation (9) which is quantitatively the dominant effect. With a lower marginal value of using $\mathbf{d}k$ in j and a delay in optimal exercise time $\partial\bar{s}/\partial z^i > 0$, the value to reallocate to j falls. In fact, $\partial^2\mathbb{C}_1/\partial(z^i)^2 > 0$; the value falls precipitously. This is the key mechanism

[11] Though I had dropped time subscripts, it is important to remember that $\bar{s}$ is a fixed number, whereas s is the state variable that is fluctuating over time.

[12] When there is no uncertainty, the value of the option is worthless; hence, there is a slight inverted U shape for $\mathbb{C}_1$ as σ^i and σ^j move away from zero. The parameter restrictions ensure that the cross-partial derivative $\partial^2\mathbb{C}_1/\partial\sigma^i\partial\sigma^j$ is monotonic.

in the paper. The complementarities between these two production inputs implicitly add a layer of indirect costs in the movement of general assets.[13]

ii. As c can be interpreted as the adjustment cost of transforming general assets for alternative use, $\lim_{c\to\infty} \mathbb{C}_1 = 0$ says that transformation cannot be prohibitively costly for reallocation to be valuable.

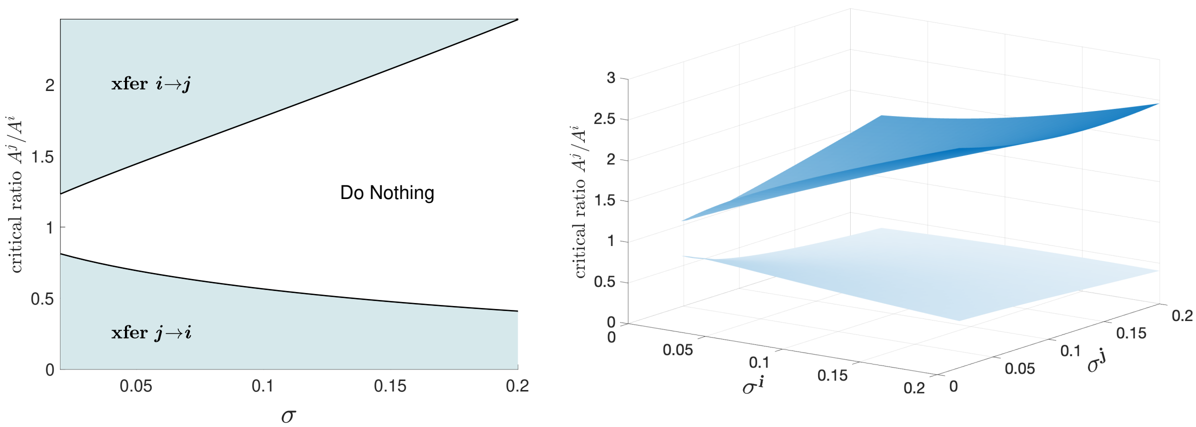
iii-v. In Figure 1, I plot the optimal $\bar{s}$ and $\underline{s}$ when volatility is symmetric and when it is asymmetric between the two product markets. With symmetry, raising the parameter value $\sigma = \sigma^i = \sigma^j$ for both businesses expands the no-action region. This is a common result in option theory and can be seen in the left panel 1a and the total derivative in (12): $\partial\bar{s}/\partial\sigma > 0$.[14] One interpretation of this result applied to reallocation is that when the volatility of each product line is high, values of the ratio s become less informative about the future trajectory of business conditions in each market. High values of σ imply the ratio s can quickly reverse, so with costly reallocation, it is optimal to delay action until it is more obvious which business the firm wants to retrench assets from and which business the firm wants to expand to.

A key difference in my model compared to the standard real options literature is that the value of reallocation is decreasing in volatility, $\partial\mathbb{C}_1/\partial\sigma < 0, \partial\mathbb{C}_2/\partial\sigma < 0$. Traditionally, when payoffs are convex in the stochastic variable, an increase in volatility increases the probability of "very good" events, so the overall option value can increase despite any delay in exercise time. In my model, the option to reallocate from $i \to j$ is implicitly a dividend-paying call option on j, where the 'dividends' are the potential gains in marginal productivity of using general assets at j instead of at i. However, it is simultaneously a short put on i when the call is exercised. Ex-ante, higher expected payoffs from an increase in the likelihood of tail events at j is offset by the simultaneous increase in the likelihood of tail events at i. All that changes is more time discounting due to a higher discount factor but the option being no closer to exercise. A symmetric argument holds for the option to reallocate from $j \to i$.

When volatility between the two product markets is asymmetric, $\partial\bar{s}/\partial\sigma^i < 0$ says that the firm is willing to reallocate sooner from i to j the riskier i is. With asymmetric volatility, it is important to think of reallocation in terms of changing the distribution of risk within

[13]With bi-directional reallocation, any change in $\bar{s}$ and $\mathbb{C}_1$ is offset by a commensurate change in $\underline{s}$ and $\mathbb{C}_2$, the threshold and value to reallocate from $j \to i$; the firm will expedite transfer from j to i if z^i increases due to the increased complementarities available in i. Though the 'net' effect balances out, I show in the next section that optimal investment in i only considers the change to $\mathbb{C}_1$ but not to $\mathbb{C}_2$ due to the value of investing in i and the value of reallocating from $j \to i$ loading on the same risk.

[14]In models of irreversible investment, this result is often referred to as the "bad news principle" due to Bernanke 1983. In fact, the ratio $\phi^+/(\phi^+ - 1)$ in Equation (10) has an analogue in Dixit and Pindyck 1994 who refer to the ratio $\beta_1/(\beta_1 - 1)$ as the shadow value of capital, or the value of assets-in-place once an investment option is exercised. It is a wedge > 1 that separates the critical value at which it is optimal to invest from the user cost of investment. To quote, "The greater is the amount of uncertainty over future values of V ... the larger is the excess return the firm will demand before it is willing to make the irreversible investment" (page 144).



(a) $\sigma = \sigma^i = \sigma^j$ (b) $\sigma^i \neq \sigma^j$

Figure 1: **Optimal Exercise Boundaries.** This figure plots the critical regions at which the firm redeploys general assets from $i \to j$ or from $j \to i$. The y-axis is the ratio of productivities between the two business lines while the x-axis is volatility of productivity shocks. Parameter values are $r = 0.035, \mu^i = \mu^j = 0.02, \rho = 0$ and $\varepsilon = 0.4, \alpha = 0.5, \theta = 1$. Moreover, $z^i = z^j = k^i = k^j = 10$ and $c = 2$ (expected reallocation time is between 3.7-5.5 years). The y-axis is $e^s = A^j/A^i$ for interpretability, where A^j corresponds to the level of productivity for product line j.

the firm because payoffs are now discounted at different rates. Recall the discount rates used to price cash flows, $\delta^i \doteq r - \mu^i + \kappa^{(1,0)}\sigma^i$ and $\delta^j \doteq r - \mu^j + \kappa^{(0,1)}\sigma^j$. If $\sigma^i > \sigma^j$ and other parameters are identical, then the reallocation option is no longer just an option to smooth cash flows, but to discount cash flows at a lower rate and offload overall firm risk by shifting production to the less volatile business. As a result, the option to reallocate from $i \to j$ is increasing in σ^i, $\partial\mathbb{C}_1/\partial\sigma^i > 0$, and is decreasing in σ^j, $\partial\mathbb{C}_1/\partial\sigma^j < 0$. A symmetric argument holds for the option to reallocate from $j \to i$.

To summarize **(i)-(v)**, for the parameters considered, comparative statics for $\bar{s}$ and $\mathbb{C}_1$ move in opposite directions; as $\bar{s}$ can be thought of as the "strike price", it is only natural that the value of an option, $\mathbb{C}_1$, is related to its moneyness. Certain primitives such as σ^i, σ^j, c are exogenous to the firm but nonetheless affect the extent to which reallocation is valuable. Others, such as z^i, are capital choices.

In the following sections, I show that changes to the primitives, by changing the levels of $\mathbb{C}_1$ and $\mathbb{C}_2$, change capital choices.

2.2 Single Investment Project

In this section, I study the decision to invest in a single project as a perpetual option.

As the baseline finding, I show that the firm's optimal investment policy can be driven entirely by the composition of an investment project. I then show limit results for values of primitives in which the composition becomes irrelevant.

Suppose the firm has a perpetual option to invest in a single investment project that produces good i. Once it is adopted, the firm's production technology in i expands from $[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}$ to $[\alpha(z^i + \mathfrak{z}^i)^\varepsilon + (1-\alpha)(k^i + \mathfrak{k}^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}$. Because the project has a defined size $(\mathfrak{z}^i, \mathfrak{k}^i)$, one interpretation of the project is the purchase of a set of production inputs. I make the following assumption about the price of a project.

Assumption 2. An investment project is a bundle of inputs set in fixed proportion $(\mathfrak{z}^i, \mathfrak{k}^i)$. The firm can acquire this bundle at price

$$\varpi^i A^i_\tau = \mathbb{E}^{\mathbb{Q}}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \Big[\alpha(\mathfrak{z}^i)^\varepsilon + (1-\alpha)(p \times \mathfrak{k}^i)^\varepsilon \Big]^{\frac{\theta}{\varepsilon}} A^i_t \mathbf{d}t \right] \tag{14}$$

where τ is the time of exercise. For analytic convenience, there are no separate factor markets for general and specific assets; instead, I assume that inputs are always bundled together as a discrete project.[15] In Appendix C.4, I relax this assumption and allow for separate factor markets with convex adjustment costs, showing that the paper's main conclusions still hold.

In Equation (14), when $p = 1$, the price of this bundle−or the cost of this investment project−can be thought as the fair value of the cash flows these assets would generate as a standalone entity, in which they obey the same production function and are subject to the same price stochasticity that the firm faces in the market for good i.

In addition to choosing $\Delta k_t^{i\to j}, \Delta k_t^{j\to i}$, the firm decides on the optimal time τ to increase production of good i via external investment. Therefore, the HJB equation (2) satisfies the four boundary conditions in (3) and (4), and two additional boundary conditions

$$H(\widehat{s}, k^i, k^j, z^i) = H(\widehat{s}, k^i + \mathfrak{k}^i, k^j, z^i + \mathfrak{z}^i) - \varpi^i \quad \text{(continuity)} \tag{15}$$

$$\frac{\partial H(\widehat{s}, k^i, k^j, z^i)}{\partial s} = \frac{\partial H(\widehat{s}, k^i + \mathfrak{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial s} \qquad \text{(optimality)} \tag{16}$$

where $\widehat{s}$ is the investment threshold. More details, including the statement of the firm's problem, is in Appendix A.3.

To emphasize the effect that reallocation has on the firm's investment policy, I first state a lemma for contrast.

Lemma 1. Suppose the reallocation of assets were not possible (for example, set $c \to \infty$). Then, the option to invest in a project is either immediately exercised or never exercised.

[15] Implicitly, I assume there is a competitive market for projects. The fair value assumption can be justified as an equilibrium price when producers with different asset compositions and different reallocation opportunities compete on prices. One can think of the bundle as a discrete asset that varies in its characteristic of specificity. The reallocation of this asset transfers the general component subject to a transformation cost and "leaves behind" the specific component as a stock.

It is convenient to consider the setting when $\varepsilon \to 0, \alpha = 0.5, \theta = 2$. Starting from $z^i k^i$, production increases to $(z^i + \mathfrak{z}^i)(k^i + \mathscr{k}^i)$ after investment but the scaled cost is only $\mathfrak{z}^i \mathscr{k}^i$, so the strictly positive cross terms $z^i \mathscr{k}^i + \mathfrak{z}^i k^i$ remain and always make it worthwhile to invest immediately.[16] With CES production, a similar reasoning applies. Complementaries between the existing stock of assets and the potential new set of assets usually make investment a "good deal".

I present the main proposition of the paper here.

Let $\mathbb{C}_1, \mathbb{D}_1$ be the integration constants corresponding to the option value of reallocating capital from i to j before and after investment as provided in Appendix A.3. Moreover, let Surplus^i denote the simple change to firm value from this project,

$$\text{Surplus}^i = \frac{[\alpha(z^i + \mathfrak{z}^i)^\varepsilon + (1-\alpha)(k^i + \mathscr{k}^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}}{\delta^i} - \frac{[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}}{\delta^i} - \varpi^i$$

Proposition 2. The optimal stopping time for adoption is the first time

$$\tau^* \doteq \inf \left\{ t : s_t \leq \frac{1}{\phi^+} \log \left(\frac{\phi^-}{\phi^+ - \phi^-} \frac{\text{Surplus}^i}{\mathbb{D}_1 - \mathbb{C}_1} \right) = \widehat{s} \right\} \tag{17}$$

Proof. See Appendix A.3. □

Moreover,

Corollary 3 (The volatility result)**.** Suppose $\rho \neq 1$ and let $\sigma^i = \sigma^j = \sigma$. There is always a level of uncertainty σ large enough such that $\tau^* = t$, regardless of its composition.[17]

To interpret Equation (17), suppose $\widehat{s}$ is a very high number−for example, if $\text{Surplus}^i \to \infty$. Then, the stopping time $s \leq \widehat{s}$ says that there is a large set of values of s for which the firm would invest. Since $s = \log(A^j/A^i)$, a value $\widehat{s} > 0$ says that the firm would acquire inputs to produce good i even if the market for i is doing poorly relative to j. In contrast if $\widehat{s} < 0$, the symmetric argument holds. The firm acquires inputs to produce good i only when i is doing well relative to j.

Together, Lemma 1 and Proposition 2 state the following. By assumption, investment projects in my model are often a good deal and sometimes a bad deal. If reallocation were not part of the firm's planning decision, the firm would either invest immediately or never invest. But adding reallocation, uncertainty over which options will be most valuable ex-post

[16] More formally, if one solves for the optimal stopping time while turning off the reallocation channel (conceptually, this is equivalent to taking the limit $c \to \infty$ which sets the value of reallocation to zero), the optimization problem is unconstrained and we have a corner solution, i.e. immediately invest.

[17] The proof takes the limit of the right-hand side of Equation 17 as $\sigma \to \infty$. After an application of L'Hôpital's rule in several places, the no-action region is an empty set and we have immediate entry into the push region.

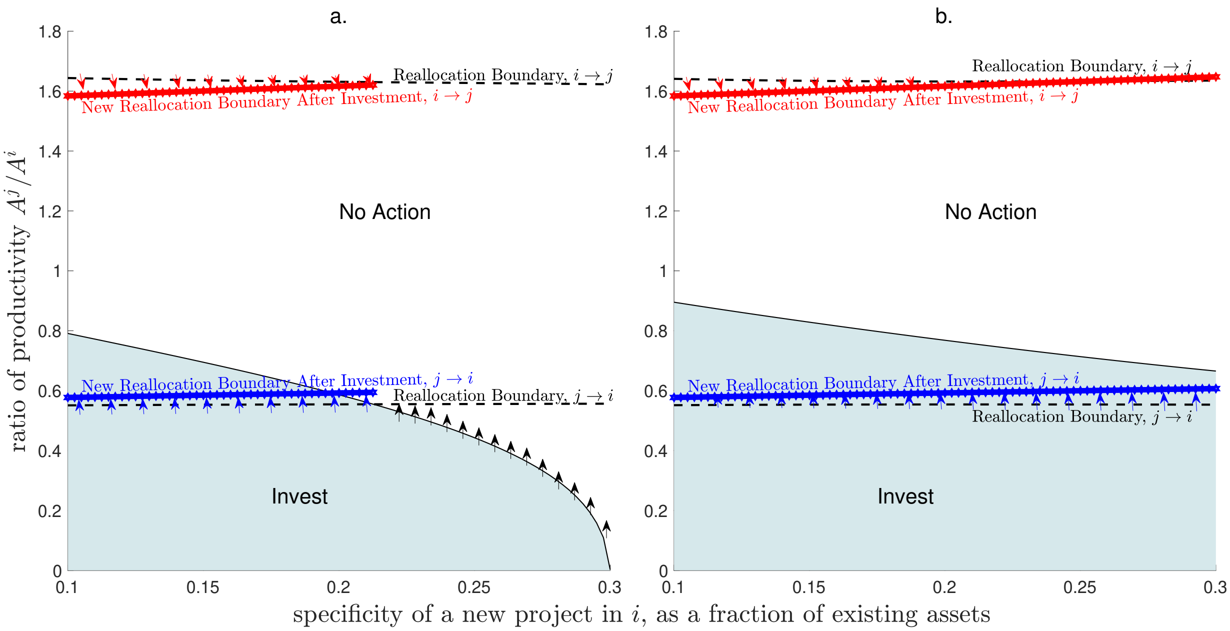


Figure 2: These figures show how varying the composition of an investment project changes the optimal boundary the option to invest is triggered. The size of general assets in this project is held fixed at 30% of the existing stock of general assets in i while the x-axis increases the size of specific assets in this project. The dashed black lines are the pre-investment reallocation boundaries and the blue and red lines are the post-investment reallocation boundaries. In the left panel a., the cost parameter p in Equation 14 is set to 1. In the right panel b., Surplus^i is held constant by varying p, so the change in optimal investment region corresponds only to the change in option values. The y-axis is $\exp s = A^j/A^i$ for ease of interpretation with CES production technology when $\sigma^i = \sigma^j = 0.08$. Other parameter values are $\varepsilon = 0.4, \alpha = 0.5, \theta = 1$. Moreover, $z^i = z^j = k^i = k^j = 10$ and $r = 0.035, \mu^i = \mu^j = 0.02, \rho = 0$ and $c = 2$.

leads the firm to delay investment in projects it would otherwise immediately invest in. Due to the costs that specific investments impose on the firm's general assets, if the firm plans to induce a large change in its asset mix via an external acquisition of production inputs, optimality requires the gains from investing to outweigh any potential loss in reallocating.

In Figure 2, I plot the optimal investment boundary while varying the composition of the investment project. The quantity of general assets in this project is held fixed at 30% of the existing stock of k^i while the specific component varies between 10-30% of z^i. For ease of interpretation, the y-axis is $\exp s$; the value on the axis corresponds to the critical ratio A^j/A^i—the relative stochastic level of profitability between the two industries—before investment occurs.

On the left, I set the cost parameter $p = 1$ in Equation (14) while on the right, I solve for p such that Surplus^i is a fixed constant regardless of the project composition. Keeping Surplus^i fixed isolates any movement in the investment boundary solely to changes in reallocation values from $i \to j$ (the denominator in Equation (17)) while the former expresses the fact that a price-taking firm might value an identical project differently based on the composition of its existing assets.[18]

The two plots show that the investment opportunities available to the firm shape the firm's planning decisions. When the firm can make a large investment in general assets with a small specific component, it invests quickly in expectation. On the other hand, if the investment project also involves a large specific component, the firm delays investment until it is no longer likely that the firm will need to move assets back to j in the foreseeable future.

The baseline finding—that a no-action region now exists—explains why firms may be hesitant to make large investments in specific, dedicated technology; these projects can handcuff the firm and compromise its ability to take other actions.[19]

However, Corollary 3 says that when there is too much uncertainty, that reallocation does not happen any sooner despite higher time discounting makes the payoffs of reallocation no longer worth preserving. Irrespective of the characteristics of the investment project, the firm immediately invests.

This is perhaps a surprising result. The choice of capital is effectively the choice of state-contingent output. One would suspect, therefore, the need to preserve flexibility is most binding when volatility is high and very bad states are more likely to occur. Instead, the result states that volatility makes waiting too costly. Since cash flows are discounted by risk-adjusted rates, postponing the benefits from investing right away becomes more difficult to stomach, while the potential benefits of exercising reallocation in the future become less valuable. Despite the increasing impetuousness of Brownian motion, the expanded size of the reallocation no-action region prevents the firm from compensating for these shortfalls by reallocating sooner and more often. Instead, these distant cash flows become relatively unimportant, freeing the investment decision from the reallocation decision.

In effect, Corollary 3 lays the groundwork for the cross-sectional predictions tested in this paper. As was pointed out at the conclusion of Section 2.1, σ^i and σ^j are parameters that the firm has little control over. Yet, they determine the value of reallocation. Importantly, high volatility makes reallocation less valuable, supported by the cross-partial $\partial^2 \mathbb{C}_1 / \partial\sigma^i \partial\sigma^j < 0$.

[18] When the composition of the investment project mirrors the composition of the firm's existing stock of assets in product market i, due to the homogeneity of the production function and the assumed price of these inputs, the net gain from investing in this project is zero.

[19] In Figure 3, I plot the 3-dimensional version of the figure, varying the composition of an investment project $(\mathfrak{z}^i, \mathfrak{k}^i)$ to be each between $10 - 30\%$, rather than keeping $\mathfrak{k}^i$ fixed. For ease of interpretation, the z-axis is $\exp s$, the critical ratio A^j/A^i before the firm invests. Since the stopping time is $\tau \doteq \{t : s_t \leq \widehat{s}\}$, the no-action region is the space above the colored plane, and the investment region is the space below it. Though the size of $\mathfrak{z}^i$ can vary further, extending $\mathfrak{k}^i$ leads to immediate investment. In Appendix Figure IA.1, I repeat the exercise, but set the volatility to be asymmetric between the two businesses. In Appendix Figure IA.2, I change the initial distribution of assets such that the firm is a "generalist" or a "specialist".

These figures have a common theme. In principle, these are mostly projects that should be immediately adopted. But given the composition of the project and the composition of the firm, the optimal placement of these investment barriers correspond to the extent to which the firm tailors its investment policy to respond to the incentive to keep alive another action within the firm.

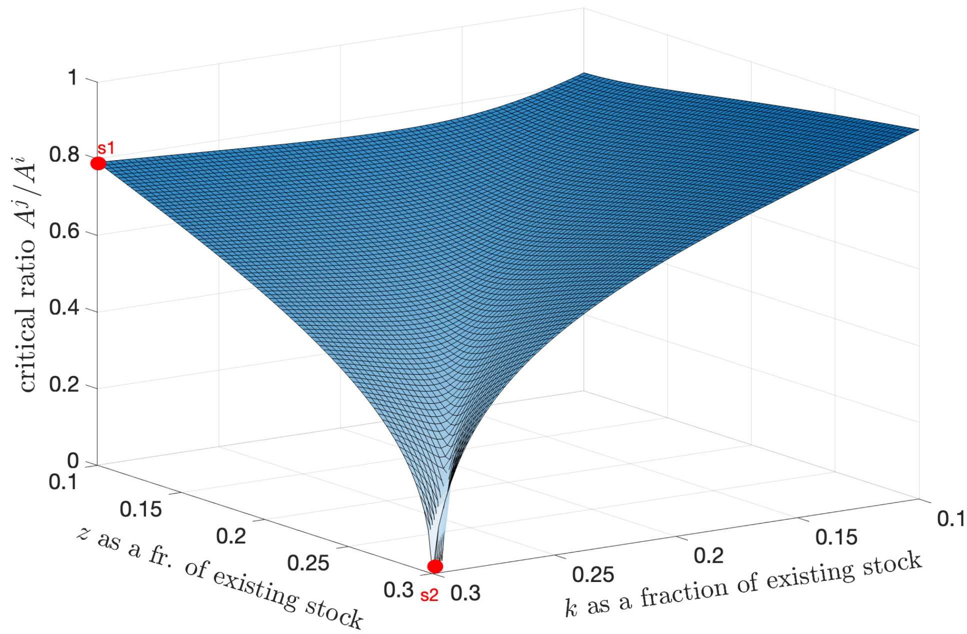


Figure 3: This figure is a 3-dimensional version of Figure 2, which is a snapshot from points "s1"→"s2" above. The left side varies the quantity of specific assets in the new project z, while the right side varies the quantity of general assets in the new project k. The option to invest is triggered for values of j/i below the plane. Parameter values are identical to those used in Figure 2.

In this section, we saw through the corollary how the comparative static flows into the investment decision. For capital planning, managers at different firms whose product lines differ in their levels of volatility should then therefore make different capital choices.

To conclude, this section focused on how *quickly* the firm invests in a project based on the incentives to preserve reallocation. In the next section, I extend the analysis to multiple project arrivals in both i and j. With multiple projects, I study investment intensity and investment specificity, and formalize the main cross-sectional predictions in the paper.

2.3 Multiple arrivals, bi-directional transfer, CRS

I start with the following assumption.

Assumption 3. Projects arrive following two separate and independent Poisson processes at rate λ^i and λ^j. A project's composition is either $(\mathfrak{z}^i, \mathfrak{k}^i)$ if the arrival is from i or $(\mathfrak{z}^j, \mathfrak{k}^j)$ if the arrival is from j, drawn independently from uniform distributions on $[\underline{\mathfrak{z}}^i, \bar{\mathfrak{z}}^i]$, $[\underline{\mathfrak{z}}^j, \bar{\mathfrak{z}}^j]$, $[\underline{\mathfrak{k}}^i, \bar{\mathfrak{k}}^i]$ and $[\underline{\mathfrak{k}}^j, \bar{\mathfrak{k}}^j]$ at the time of arrival. Moreover, a project expires if it is not adopted by the arrival of the next project. Its price follows the structure in Assumption 2.

Importantly, both the composition of a project and the maturity of an investment option are random, so the firm's only decisions are again whether to reallocate and whether to adopt an incoming project. Random project composition conveys the idea that the firm's preferred type of investment projects may not always be available; it only has the choice to invest in it. Expiration of an investment option conveys a timing window and urgency; in contrast to the perpetual option case in which all projects are eventually adopted after a long enough wait, project expiry allows investment projects that do not meet the current needs of the firm to be skipped entirely. After multiple project arrivals, the cumulative effect of skipping a particular type of project shows up in the average characteristics of projects that *are* adopted through the average ratio $\mathscr{k}/\mathscr{z}$.

I show in the proposition below that the optimal boundary satisfies an equation that can be represented for project i,

Proposition 3. The optimal $\hat{s}^*$ satisfies

$$
\begin{aligned}
&\phi_1^- \times \text{Surplus}^i \\
&\quad = (\phi_0^+ - \phi_1^-)\Bigg[\frac{\lambda^i}{\lambda^i+\lambda^j}\Big(\mathbb{D}_1^i(k^i+\mathscr{k}^i+\tilde{\mathscr{k}}^i,k^j) - \mathbb{C}_1^i(k^i+\tilde{\mathscr{k}}^i,k^j)\Big) \\
&\qquad\qquad + \frac{\lambda^j}{\lambda^i+\lambda^j}\Big(\mathbb{D}_1^j(k^i+\mathscr{k}^i,k^j+\tilde{\mathscr{k}}^j) - \mathbb{C}_1^j(k^i,k^j+\tilde{\mathscr{k}}^j)\Big)\Bigg] e^{\phi_0^+\hat{s}^*} \\
&\quad + (\phi_0^- - \phi_1^-)\Bigg[\frac{\lambda^i}{\lambda^i+\lambda^j}\Big(\mathbb{D}_2^i(k^i+\mathscr{k}^i+\tilde{\mathscr{k}}^i,k^j) - \mathbb{C}_2^i(k^i+\tilde{\mathscr{k}}^i,k^j)\Big) \\
&\qquad\qquad + \frac{\lambda^j}{\lambda^i+\lambda^j}\Big(\mathbb{D}_2^j(k^i+\mathscr{k}^i,k^j+\tilde{\mathscr{k}}^j) - \mathbb{C}_2^j(k^i,k^j+\tilde{\mathscr{k}}^j)\Big)\Bigg] e^{\phi_0^-\hat{s}^*} \\
&\quad + (\phi_1^+ - \phi_1^-)\left[\mathbb{D}_1(k^i+\mathscr{k}^i,k^j) - \mathbb{C}_1(k^i,k^j)\right] e^{\phi_1^+\hat{s}^*}
\end{aligned}
\tag{18}
$$

which can easily be solved numerically for $\hat{s}^*$, and written similarly for project j in Appendix Equation A29. Moreover,

$$
\begin{aligned}
\phi_0^\pm &= \frac{-\tilde{\mu} \pm \sqrt{2\tilde{r}\tilde{\sigma}^2 + \tilde{\mu}^2}}{\tilde{\sigma}^2} \\
\phi_1^\pm &= \frac{-\tilde{\mu} \pm \sqrt{2(\tilde{r}+\lambda^i+\lambda^j)\tilde{\sigma}^2 + \tilde{\mu}^2}}{\tilde{\sigma}^2}
\end{aligned}
$$

and $\tilde{\mathscr{z}}^i, \tilde{\mathscr{z}}^j, \tilde{\mathscr{k}}^i, \tilde{\mathscr{k}}^j$ denote the expected values of the random draws.

Proof. See Appendix A.4. □

With multiple project arrivals, the two options−reallocation and investment−are two levers for the firm to manage risk and to grow. The proposition states that the firm actively

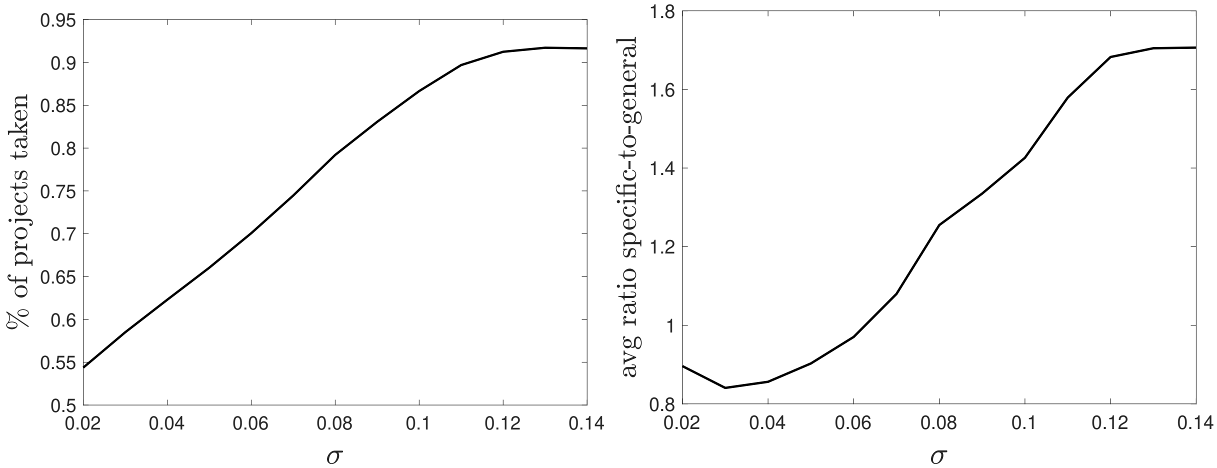


Figure 4: **Investment and Specificity.** This figure presents the main predictions of the model. The left plots overall investment while the right plots the average specificity of investment projects as functions of volatility. These results come from simulations pooled across 5,000 sample paths over 10 years. The x-axis varies the volatility of both product markets $\sigma = \sigma^i = \sigma^j$. On the left, the y-axis is the average percent of available projects that are taken by the firm in a sample path, over all sample paths. On the right, the $y-$axis is the average ratio, $\varkappa/\Bbbk$, of these projects that are taken. Parameter values are $r = 0.035, \mu^i = \mu^j = 0.02, \rho = 0$ and $\varepsilon = 0.4, \alpha = 0.5, \theta = 1$. Moreover, the initial values are $z^i = z^j = k^i = k^j = 10$ and $c = 2$, and $\lambda^i = \lambda^j = 0.25$, where the project composition $\mathfrak{z}$ and $\Bbbk$ are each independently drawn from a uniform distribution on $[0.005, 0.095]$ as a percentage of the stock of capital at t.

manages the two levers to preserve flexibility while expanding capacity; in doing so, the firm makes sure it can always access the option—whether moving assets internally or acquiring them externally—that most efficiently meets its current needs.

To illustrate the effect of volatility on the firm's optimal investment rate and the specificity of its projects, I simulate 5,000 sample paths over 10 years. For symmetry, I vary $\sigma = \sigma^i = \sigma^j$, and simulate over a short 10 year horizon, and opt for frequent, small projects over infrequent, large ones to net out the influence of random draws. I set $\lambda^i = \lambda^j = 0.25$, which averages to about 6 projects a year, and the average size of a project to be 5% of the capital stock at t. The firm reallocates capital whenever $s > \bar{s}$ or $s < \underline{s}$.

Figure 4 plots two important statistics relating to investment in these simulations. In the left panel, I plot the average percent of available projects invested in, across all sample paths. In the right panel, I plot the average ratio, $\varkappa/\Bbbk$, pooled across all projects that are adopted. The $x-$axis is the level of volatility in each market. The two plots show that when σ is low, the firm accumulates more general assets on average because it delays and eventually skips investments in projects that are too specific; $\varkappa/\Bbbk$ is a measure of specificity. However as σ increases, there is recovery in both the investment rate and average specificity of projects.

Figure 4 confirms the intuition derived in the previous sections. "Risky" firms—firms whose product lines face constantly high volatility—cannot afford to strategize around events that may occur several years down the line. Investment projects today, no matter how specific, are too valuable to pass up. On the other hand, less risky firms, because they can afford to wait, delay investment until the composition of the project matches the objectives of the firm. The main empirically testable predictions are:

H1. Investment rates are increasing in firm-level risk

H2. Average specificity of investment is increasing in firm-level risk

An important consequence from the model in addition to the main predictions is that cross-sectionally, there is a positive relationship between investment and uncertainty; after all, less volatile firms are squeamish toward one half of the set of production inputs.

In Appendix C.4, I study the case in which the firm can adjust k^i, z^i, k^j, z^j each separately. As long as the cost of reallocation within the firm is not strictly greater than the marginal cost of adjusting capital externally via investment, reallocation is valuable to the firm. But due to the constant returns to scale and CES production function, the firm will at one point stop increasing its stock of general assets in one business until it adds more specific assets. The same logic would then carries over, showing that these results are not unique to the assumption of bundling the inputs together.

In Appendix C.1, I study additional scenarios in which the initial stock of general and specific assets differ such that the firm starts out as a "generalist" or a "specialist". The simulations show that while there are differences, the differences are minor. Rather, the level of volatility has the dominating effect. In Appendix C.2, I endogenize the choice to expand from a one-product firm to a two-product firm and show that the choice to expand depends on complementarities.

Finally in Appendix C.3, I introduce stochastic volatility in the model using a two-state Markov regime switching process. When short periods of high volatility are followed by prolonged periods of low volatility (as is the case in the data), an uncertainty shock today implies that the option to reallocate is expected to be exercised in the low volatility regime. As a result, the discount rate used to price the reallocation payoff is lower when the firm is in a high volatility regime due to the likelihood of the regime change. In turn, the option value is higher. The same economic mechanisms then carry over: if reallocation is valuable, it is worthwhile to delay or skip investment projects to preserve flexibility that might be needed in the future.

In contrast to the cross-sectional result via comparative statics that average investment is positively associated with firm-level uncertainty, stochastic volatility introduces a time-series component: within the same firm, an arrival of a high-volatility regime leads to an immediate decrease in investment and a decrease in the average specificity of investment. Moreover, given the assumption that periods of low volatility last longer than periods of high volatility, firm-level uncertainty can be separated into short-term and medium/long-term components to separate the two effects.

2.4 Asymmetric Volatility

In this section, I refine the main findings and study the case when volatility is asymmetric between the firm's two product lines. This setting maps well to multi-segment firms who operate in distinct sectors. The main mechanism and intuition built from the previous sections carry over, and I make an empirical prediction on the within-firm investment policy for multi-segment firms. Moreover, I derive semi-closed solutions for investment intensity directly (with some simplifying assumptions).

Suppose that at each instant, the firm can acquire a discrete set of production inputs to produce good i. Each bundle pairs together a mix of general and specific assets and is priced as before following Assumption 2. The expected size of the project is set at fixed proportions of the firm's current capital stock.

If the firm does not purchase this bundle, the project "expires" and a new project arrives the next instant. Projects are take-it-or-leave-it offers, similar to the setup in Gomes et al. 2003. With this structure, the firm will invest as long as the net present value of investing is non-negative, delineated by the region $s_t \le \widehat{s}$. If one project is available in each unit of time, the cumulative travel time for the stochastic process $s_t = \log(A_t^j/A_t^i)$ to hit N consecutive investment barriers gives a natural characterization of the firm's investment intensity.

For simplicity, let reallocation only occur in one direction from $i \to j$. Moreover, let $\widehat{s}_n$ denote the investment threshold for the nth investment project.

Proposition 4. Define I_N as the investment rate to complete N projects. Then,

$$I_N = N \Bigg/ \sum_{n=1}^{N} \left[\frac{\widehat{s}_n - \widehat{s}_{n-1}}{\mu^s}, 1 \right]^+$$

where $[a,b]^+ \doteq \max\{a,b\}$. If $[(\widehat{s}_n - \widehat{s}_{n-1})/\mu^s, 1]^+ > 1 \ \forall n$,

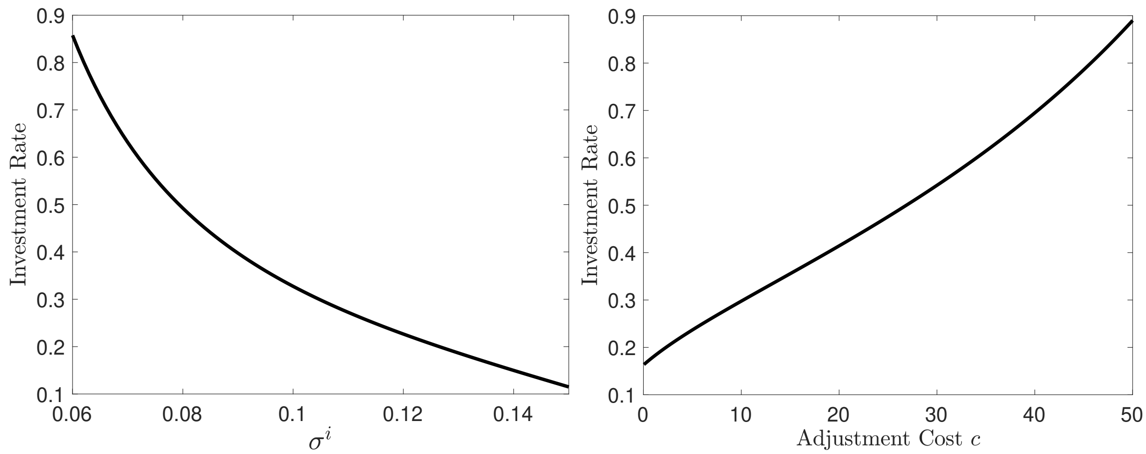
$$I_N = N \Bigg/ \left(\frac{\widehat{s}_N - s_0}{\mu^s} \right) \tag{19}$$

where s_0 is the initial position of s_t, $\mu^s = \mu^j - \mu^i + \frac{1}{2}\left((\sigma^i)^2 - (\sigma^j)^2\right)$ by Itô's lemma, and $\widehat{s}_n$ for $n = 1, ..., N$ given in Appendix A.5.

Proof. See Appendix A.5. □

Moreover,

Corollary 4. $\lim_{c\to\infty} I_N = 1$



(a) $\sigma^j = 0.06$ and $\sigma^i \geq 0.06$

(b) $\sigma^j = 0.06$ and $\sigma^i = 0.14$

Figure 5: **Expected Investment Rate.** Computes the expected investment rate in business i using Equation (19) to complete $N = 25$ projects. Parameter values are $\mu^i = .04, \mu^j = 0.02, \rho = 0$. Moreover, σ^j is fixed at $\sigma^j = 0.06$. The left figure varies σ^i for $c = 2$. The right figure fixes $\sigma^i = 0.14$ and $\sigma^j = 0.06$ while varying c. Initial parameters are set to $z^i = k^i = 15$ and $z^j = 25$ and $k^j = 5$.

For all intent and purposes, Equation (19) says the investment rate can be characterized simply by the distance between the initial point s_0 and the final investment threshold $\widehat{s}_N$. Corollary 4 is the multiple project analogue of Lemma 1 which states that the firm has no reason to postpone investment if reallocation were not in its cards.

In Figure 5, I plot the expected investment rate in business i when $\sigma^i > \sigma^j$. On the left, Figure 5a varies σ^i along the x-axis while on the right, Figure 5b varies the adjustment cost c while holding σ^i fixed.

The left plot shows that as σ^i increases while holding σ^j, μ^i, μ^j fixed, the investment rate in i is monotonically decreasing. There are two reasons. The first is mechanical; as σ^i increases, the drift μ^s tends towards zero. The second reason follows the logic in Section 2.1. The option to reallocate assets to a business in which cash flows are discounted at a lower rate is very valuable to the firm. Hence, the firm avoids accruing impediments to offload production and is willing to skip investment projects in i until it becomes unlikely the firm will need to make use of the option to reallocate to j.

The right plot shows that this effect is subdued when the adjustment cost is too large. In the model, I interpret c as the cost of transforming general assets for alternative use. When the cost is small, the firm is willing to reallocate assets sooner and more often. When the cost is prohibitively high, the firm moves assets only as a last resort. The figure shows that when the adjustment cost is too high, the value of reallocation is negligible so it does not actively plan investment around preserving this secondary channel.

In Appendix Figure IA.3, I show how the investment rates change for different parameter combinations of $\alpha, \varepsilon, z^i, z^j, k^i, k^j$.

Proposition 4, Corollary 4, and the two plots convey conditional tests of the predictions H1 and H2.

H1a. Multi-segment firms invest relatively less in their riskier segments, but this effect is offset the more unrelated the segment is to the firm's other businesses.

H2a. If reallocation is not costly (or prohibitively costly), there are no cross-sectional differences in firm-level specificity as a function of uncertainty.

3 Description of Data

3.1 Transition to Empirical Analysis

Investments in specific, hard-to-reverse technology impose a cost on the firm's existing set of general assets. In the baseline finding in Section 2.2, I showed that due to this interaction, the firm evaluates projects on a case-by-case basis based on the composition of the project and the composition of the firm. The comparative statics and limit results examined how changes to "primitive" parameters that are exogenous to the firm changed capital choices. In Section 2.3, I formalized the intuition derived from a single investment project to multiple project arrivals. In Section 2.4, I refined the main result to asymmetric volatility, which is applicable to multi-segment firms that pair together businesses in distinct sectors.

The cross-sectional predictions generated by the model, re-stated in Section 4, relate the model primitives, volatility and adjustment costs, to capital choices−investment intensity and specificity. In this section, I discuss construction of all four variables.

Detailed data definitions for other control variables used in the empirical analysis are in Appendix Table B1.

3.2 Financial Data

3.2.1 Uncertainty

The literature on investment under uncertainty and asset specificity traditionally measures firm-level uncertainty using annualized realized volatility from daily stock returns over 252 trading days. See Leahy and Whited 1996, Bloom 2009, Gilchrist et al. 2014, Gulen and Ion 2016, Kim and Kung 2016, Kermani and Ma 2023 and Alfaro et al. 2024.

I follow a similar approach but extend the rolling window up to 7 years. The difference is due to focus; this paper is not about the firm's investment response to short-term uncertainty shocks. As the baseline measure, I use a 5-year window, intended to capture the perceived business risk by a manager that is relevant for medium to long-term objectives of the firm. In Appendix Table IA.1, I show the average volatility of volatility for firms in the merged

CRSP-Compustat data. The standard deviation of firm-level uncertainty for each firm, averaged across all firms, roughly halves across the 5th, 25th, 50th, 75th and 95th percentile distributions when changing from 1 year to 5 year windows; the median standard deviation within a firm is 7-8% while the average is 10-11%. This suggests that the 5-year measure is likely slow moving enough to capture the intended perceived medium to long-run risk.

Furthermore, I supplement this measure of medium to long-run volatility by de-levering this series by 1 plus the firm's gross debt-equity ratio. This follows the textbook relation that beta on unlevered equity is the beta on assets.[20] Following Gilchrist et al. 2014 and Kermani and Ma 2023, I also include the volatility of residuals from a regressions of excess returns on the Fama-French factors;[21] I use rolling regressions over 5-year windows and use 5-factors per Fama and French 2016. All measures are as of the end of June for a calendar year to standardize across firms. Moreover, I lag by 1 year, using windows from $t-5$ to $t-1$.[22] I require stocks to have at least 230 observations of non-missing data each calendar year, with share code $10, 11$ and exchange code $1, 2, 3$. For the sake of robustness, I also include the uncertainty measure using monthly data instead of daily, and compute the standard deviation of sales over 5 years scaled by lagged assets using quarterly data; however one should be mindful that these measures use 60 and 20 data points respectively.

The first prediction in my model assumes symmetric volatility between the firm's businesses. When a firm operates in a single industry, I make the modest assumption that the volatility of cash flows is likely to be "close enough" across its major product areas such that my modeling assumption is satisfied. For the third prediction, which investigates the case when volatility is asymmetric, I study multi-segment firms using Compustat segments. For segment-level volatility, I proxy this measure using industry averages of my 5-year volatility measure. I primarily use 2-digit SIC codes, using the Compustat variable "sich" for industry definitions.

3.2.2 Investment Rates

To test the first prediction, I construct the investment rate in physical, human, and knowledge capital. Following Belo et al. 2014, for physical capital investment, the investment rate is capital expenditures divided by the average of current and lagged net property, plant and equipment (PPE): $I/K_t = I_t/(.5K_{t-1} + .5K_t)$. Moreover, the investment rate is bounded between $[-0.5, 0.5]$ to remove the influence of outliers. For investment in human capital, I use net hiring rates measured by net hiring divided by the average of current and lagged

[20] For example, Berk and DeMarzo 2016. Implicitly, this method assumes the beta on debt is equal to 0.

[21] To quote Gilchrist et al. 2014, "... is an estimate of time-varying equity volatility for firm i, a measure that is purged of the forecastable variation in expected returns ...".

[22] Investment rates at $t+1$ use data variables measured at t. This means that stock price data used to compute volatility may incorporate up to one half of capital decisions in fiscal year t.

total employment: $\Delta H_t = (H_t - H_{t-1})/(.5H_{t-1} + .5H_t)$, where H_t is total employment at t.

To construct investment rates for knowledge capital, I first construct the stock of knowledge capital using the perpetual inventory method following Peters and Taylor 2017 and Belo et al. 2022 which I describe in detail in Appendix B.2. The corresponding investment rate is: $RD/K_t^K = RD_t/(.5K_{t-1}^K + .5K_t^K)$ where K_t^K is the stock at time t, and bounded above at 0.5. For robustness, I also study changes in flows to R&D expenditures and intangible investment, Peters and Taylor 2017, for firms active in R&D. Following Alfaro et al. 2024, $\Delta x_t = (x_t - x_{t-1})/(.5x_{t-1} + .5x_t)$, which is identical to the construction of ΔH_t. This definition of growth rates has the appealing properly that growth rates are bounded between -2 and 2. Since R&D write-offs are not included as investments in capital stock but are included as expenses, there are slight differences in these data series.

Following the literature, I exclude firms with missing values for assets or sales, and firms in utilities (2-digit SIC code 49), finance (60-69), and government (90+).

For the third prediction, which studies investment in segments that differ in risk, I use data from Compustat segments. I define a multi-segment firm at the 2-digit SIC level and focus on firms with at least two divisions. At the segment level, data for capital expenditures is well populated but net PPE is not, so I define the segment investment rate as segment capital expenditures divided by the firm's net PPE for that fiscal year. Moreover, for investment in human capital, I impute division hiring rates using ΔH_t scaled by the segment's weight in sales.[23] Since these measures may skew towards which division the firm wants to grow, I include a battery of controls which I describe in Section 4. For robustness, I also scale by segment identifiable total assets. Following Denis et al. 2002, I exclude firm-year observations in which the sum of division sales and total sales in Compustat differ by more than 1%. Moreover, I exclude firms with at least one division in utilities, finance, and government as before, and following Boguth et al. 2022, I exclude firms with at least one division in agriculture. I require firms to have a valid stock price with share code $10, 11$ and exchange code $1, 2, 3$. All control variables are industry averages using firm variables from the main Compustat data.

3.3 Job Postings

I divide this subsection into three parts. First, I discuss the data and show a sample job posting along with summary statistics on the most frequently demanded skills. In 3.3.2, I describe my measure of specificity. It uses each firm's job postings over a calendar year to quantify a firm's relative demand for generalist workers. The measure is a proxy for the ratio $\mathcal{k}/\mathcal{z}$ in the model, which showed that firms prioritizing reallocation invest more in general

[23]For multi-segment firms, employee by division is populated by less than 20%. Hence, imputing values is necessary. The literature on conglomerates use division sales as the appropriate weighting variable.

projects on average.

Moreover, I discuss proxies for the adjustment cost c for multi-segment firms by quantifying the degree of relatedness between the firm's segments. I measure relatedness using the overlap of specialized worker skills, and it is intended to capture the extent to which segments share a common set of production inputs vis-à-vis workers whose expertise are useful in other areas within the firm. In my model, I interpret the adjustment cost of moving general assets as the cost of transforming these production inputs for alternative use. The size of the costs, in turn, affects the value of reallocation opportunities. Conceivably, this cost of transformation is likely to be smaller if two segments are closely related than if they are very unrelated.

In 3.3.3, I address potential concerns of using job postings data in relation to my model, and present supporting statistics.

3.3.1 Description

To construct measures of specificity, I use online job postings data from Lightcast (formerly Burning Glass). Data is from 2010-2023, and includes approximately 270 million unique postings. Figure 6 shows select portions from a sample job posting. As a separate data variable, Lightcast lists the specialized skills inferred from the body of the text. These skills are extracted from the required & minimum qualifications when available and from the job description itself. The skills are then normalized under Lightcast's own taxonomy. Specialized skills are task-based, and are the "hard" skills needed to fulfill the job description. They are distinct from "soft" skills such as teamwork, leadership, etc, which Lightcast categorizes under "Common Skills"[24].

In Appendix Table IA.2, I show 50 skills with the highest total frequency over this sample period. "Project Management" tops the list, followed by "Marketing", "Nursing", "Auditing" and "Accounting". SQL ranks #11, Java at #25 and Python at #41. These skills are demanded in every industry, defined at the 2-digit SIC code. Perusing through this list, some skills are so broadly defined that they lack any meaning; but overall, all skills on this list relate to tasks that are necessary for corporations to function. For example, the surprising entry of "Forklift Truck" at #34 can be explained by the fact that any company that has a warehouse needs someone who can operate a forklift.

Since the goal is to construct firm and industry level measures of specificity, I initially exclude job postings for positions in administrative, middle and back office roles.[25] One worry

[24] Deming and Noray 2020 also use this data variable, but to study wage inequality, skill obsolescence and the rate of skill change across different occupation categories.

[25] I filter out these roles using Lightcast's occupation taxonomy (similar to the Bureau of Labor Statistics' Standard Occupational Classification codes). As a baseline, I remove clerical & administrative roles (LOT Career Area,12), Finance (19), Human Resources (22) and Law, Compliance, and Public Safety (24).

Company: Apple Inc.
Job Title: DSP Audio Engineer
Posting Date: 2024-08-15
Location: Austin, TX

Description: Our team designs and implements audio processing IP to meet the needs of Apple's current and future products. In this hands-on engineering role you will contribute to the complete engineering chain from user needs discovery and algorithm development to the design, modeling and debugging of audio signal processing hardware. As part of the SoC Architecture team we digest product and software requirements and provide architecture specs for audio IP to our Silicon Engineering teams. We also provide C / MATLAB models for these audio IP. Additionally, our team is involved in performance modeling and analysis of audio workloads and the appropriate sizing of audio subsystems in our SoC's. We have visibility across product lines and are responsible for building scalable architectures that can be widely deployed. We work closely with teams across Platform Architecture, Silicon Engineering, Software, Hardware Engineering and others throughout the lifecycle of a chip from early exploration, to execution, to tape-out, to shipping a product.

Minimum Qualifications BS/MS/PhD in Computer Science, Electrical Engineering, or related field required

Preferred Qualifications - Computer Architecture (CPU, DSP, ML accelerator etc.) - Digital Signal Processing, audio experience preferred - C / C++ knowledge - Python / Perl - Fixed point / Floating point knowledge - MATLAB - RTL Design - HW/SW co-design - SoC Power modeling - SoC architecture concepts - clocking, on-chip interconnects etc. - Machine Learning

Lightcast Specialized Skills:
C (Programming Language), C++ (Programming Language), Debugging, Audio Engineering, Perl (Programming Language), Electrical Engineering, Digital Signal Processing, Audio Signal Processing, Algorithm Design, Computer Engineering, Python (Programming Language), Floating Point Algorithm, Machine Learning, Computer Architecture, Scalability, Register-Transfer Level, MATLAB, Performance Modeling, Computer Science

Figure 6: **Sample Job Posting.** The above figure shows select portions from one of Apple Inc.'s job posting in the Lightcast IO database. "Lightcast Specialized Skills" corresponds to a list of skills extracted from the body of the text, using Lightcast's propriety algorithm to classify required or recommended expertise and competencies under its standardized taxonomy. Lightcast encodes these skills in the data variable: "SPECIALIZED_SKILLS_NAME". Over the sample, there are more than 30,000 uniquely identified specialized skills.

is that large corporations in the U.S. have legal, finance and human resources divisions, as well as security and building maintenance. But increased demand for support staff do not correspond to investment in general capital that can potentially be reallocated for alternative use. Later in the empirical analysis, I show that results minimally change when I add back these positions.

Otherwise, I consider all postings between 2010-2023 for full-time job positions in which which the company has an identified industry code and a valid company name.

3.3.2 Measures of Specificity

Using standard techniques in textual analysis, I use the specialized skills associated with each job posting to identify skills and expertise that are most important and unique to an industry. In short, I apply the 'term-frequency-inverse-document-frequency' (tf-idf) algorithm to weight skills based on how often they appear across all postings in a given industry-year, then down-weight them by how many other industries ask for these skills.

In a change of notation, I use s to denote a skill, j for industry and i for firm. For brevity, I drop time t subscripts, but all measures are constructed at an annual frequency using job postings posted in that year.

Industry

To construct industry measures of specificity, I aggregate upwards at the 2-digit SIC level. The tf-idf for skill s in industry j is

$$\text{TFIDF}_{sj} \doteq \text{TF}_{sj} \times \text{IDF}_s$$

where

$$\text{TF}_{sj} \doteq \frac{s_j}{\sum_p s_{pj}}$$

measures the relative importance of skill s in industry j. The numerator s_j is the number of times skill s is posted in industry j, and $\sum_p s_{pj}$ is the total number of skills posted across all postings in industry j. Moreover,

$$\text{IDF}_s \doteq \log\left(\frac{\text{total number of industries}}{\#\text{ industries that asked for } s}\right)$$

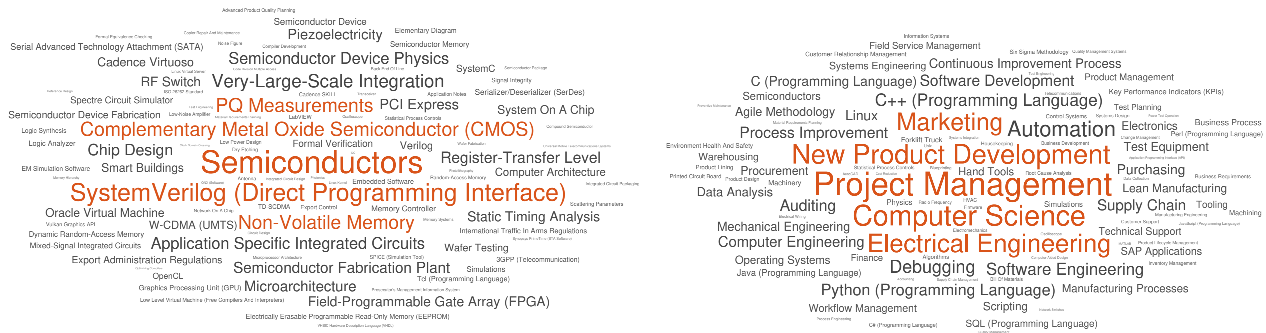
When a skill appears in almost every industry, the idf term approaches 0. Therefore, the tf-idf approach emphasizes skills that appears frequently in industry j, but not frequently elsewhere; it is a natural weighting method to identify "specific" skills. As a contrast, I construct the set of "general" skills using $\text{TF}_{sj}/(1 + \text{IDF}_s)$.

For each industry, the vector V_j is a collection of TFIDF_{sj} normalized to have unit length,

$$V_j = \frac{\text{TFIDF}_{sj}}{||\text{TFIDF}_{sj}||}$$

Figure 7 shows specific and general skills using the full sample of job postings by firms in 2-digit SIC code 36: Electronic & Other Electric Equipment. The word clouds show that the tf-idf weighting method accurately identifies the set of skills and expertise that are "specific" to an industry. The left plot relates to semiconductors while the right plot relates to engineering and programming.[26]

[26]Since this writer is more familiar with finance than semiconductors, in Appendix Figure IA.4, I show a companion figure for 2-digit SIC code 62: Security & Commodity Brokers, which include specific skills such as "OTC Derivatives", "Quantitative Investing", "Wealth Management" & "Life Insurance Underwriting".



(a) Specific Skills

(b) General Skills

Figure 7: **Skills in sic−36: Electronic & Other Electric Equipment.**

Firm

To match to firms in Compustat, I use Lightcast's company-to-gvkey key, and supplement this list with fuzzy matching on firm names from the merged CRSP-Compustat database.

For firm i, I define

$$F_i = \frac{\text{TFIDF}_{si}}{||\text{TFIDF}_{si}||}$$

where $\text{TFIDF}_{si} = \text{TF}_{si} \times \text{IDF}_s$ uses the firm's own postings, and IDF_s is the same as before. The TF_{si} term emphasizes skills that most frequently appear in the job descriptions solicited by the firm.

In Appendix Figure IA.5, I show the word cloud for the highest weighted skills in Ford Motors' tf-idf vector F_i using job postings from 2023, which notably feature specialized skills such as "Autonomous Cruise Control Systems"; this skill is consistent with Ford's emphasis on the development of L3 driver assistance systems in its 2023 earnings call transcripts.

Let $\mathscr{I}$ denote the set of 2-digit SIC codes, and let j denote a firm's primary industry at year t. I construct the following measure of specificity,

$$\text{General}_i = \frac{1}{|\mathscr{I} \backslash \{j\}|} \sum_{k \in \mathscr{I} \backslash \{j\}} F_i \cdot V_k \tag{20}$$

defined as the average cosine similarity of the firm's tf-idf weighted skill vector against all industries outside of the firm's own industry j at year t. In most cases, there are 82 sic-2 codes in this sample so $|\mathscr{I} \backslash \{j\}| = 81$, but as additional checks I include all businesses reported in Compustat segments in fiscal year t when available for multi-segment firms and Compustat's primary industry when Lightcast and Compustat disagree.[27]

[27] Let $\mathscr{J}$ denote the set of industries in which the firm operates, taking the union of Lightcast's company industry classification, Compustat variable "sich" and industries reported in Compustat segments. The measure is then appropriately modified over $\mathscr{I} \backslash \mathscr{J}$.

By construction, this measure quantifies the extent to which the firm desires workers with skills and expertise that are useful in other industries. It measures "how general". A firm has a low score when on average, it predominantly asks for a precise set of skills that do not appear elsewhere; this is almost by definition describing something that is "specific". On the other hand, a firm that prioritizes flexible workers is likely ask for broadly defined skills that overlap with skills possessed by workers in other industries. It is a proxy for the statistic $\mathscr{k}/\mathscr{z}$ from Section 2.3. In the same way that a firm that prioritizes reallocation in the model invests in projects with a higher average ratio $\mathscr{k}/\mathscr{z}$, firms that desire generalist workers have a high score for General_i.

Within-Firm

Using each industry's tf-idf weighted vector V_j, I measure the relatedness between any two industry pairings by the cosine similarity of their skill vectors. Since each industry's skill vector weights skills based on how frequently a skill appears in job postings within its industry and how infrequently it appears in others, a high cosine similarity between two industries indicates that their important and unique set of skills overlap. The cosine similarity score between industry j and industry k is

$$\rho_{j,k} = V_j \cdot V_k$$

$\rho_{j,k} \in [0, 1]$ where complete overlap is $\rho_{j,k} = 1$ and no overlap is $\rho_{j,k} = 0$. I compute this score at the annual frequency and pooled across all years from $2010-2023$. In Figure IA.6, I show the relatedness for select industries (sic-2 codes 10-40) for the full sample of job postings from 2010-2023.

Let $\mathcal{J}$ be the set of industries in which a firm operates via Compustat Segments at the 2-digit SIC level. For segment j (where j corresponds to the segment's industry code), its $\text{Relatedness}_{i,j}$ is the average cosine similarity score between industry j and all other industries in which the firm operates,

$$\text{Relatedness}_{i,j} = \frac{1}{|\mathcal{J}\backslash\{j\}|} \sum_{k}^{\mathcal{J}\backslash\{j\}} \rho_{j,k} \tag{21}$$

When a segment is very similar other businesses within the firm, the relatedness score is high. I interpret this to be that the adjustment cost of moving general workers out of this segment is *low*.

To illustrate this concept, in Figure 8, I plot the skills that have the highest element-wise products in $V_j \circ V_k$ for two sample industry-pairings; this allows us to see the skills that feature most prominently in the calculation of $\rho_{j,k}$. On the left panel (a), I plot the skills

(a) Chemicals & Allied Products (28) and Measuring, Medical & Optical Equipment (38)

(b) Apparel & Other Textile Products (23) and Leather & Leather Products (31)

Figure 8: **Overlapping skills with the highest weights in $V_j \circ V_k$.**

that are highly weighted between 2-digit SIC codes 28 (Chemicals & Allied Products) and 38 (Measuring, Medical & Optical Equipment). On the right panel (b), I plot for 23 (Apparel & Other Textile Products) and 31 (Leather & Leather Products). According to Figure IA.6, these industries have a relatively high overlap ($\rho_{28,38} = 0.28, \rho_{23,31} = 0.62$). In (a), many of the overlapping skills and expertise relate to medical devices, clinical trials and regulations while in (b), they relate to garment design, especially relating to footwear.[28] Notably, the most important skills from 28 (Chemicals & Allied Products) relating to biochemistry, pharmacology, and expertise in various diseases are missing.

This omission shows up in the difference in the degree of relatedness for the two pairs given $\rho_{28,38} = 0.28$ and $\rho_{23,31} = 0.62$. Whereas footwear design features prominently in both 23 and 31, panel (a) suggests that there is a much lower limit to the extent workers who work in these two industries—for example, employees in Johnson & Johnson—can seamlessly transition between the two; it is plausible, therefore, that the adjustment costs of moving workers between 28 and 38 is likely to be larger than between 23 and 31.

3.3.3 Two Concerns

I address two potential concerns. The first is that job postings do not equal job hirings. However, Bloom et al. 2021 show statistics that job postings on Lightcast move one-to-

[28]According to Lightcast: "Honeywell Operating System (HOS) is a specialized skillset that ... enables users to monitor and control various processes and systems such as temperature, pressure, flow, and level control. Honeywell operating system professionals must have a deep understanding of hardware and software development, real-time operating systems, programming languages, and industrial control systems. They play a critical role in ensuring the safe and efficient operation of various industries, including energy, chemical, and manufacturing."

one with the job postings in the U.S. Bureau of Labor Statistics' Job Openings and Labor Turnover Survey (JOLTS). Moreover, they find that the volume of job postings for a given job category in Lightcast corresponds to actual employment in that job category per Occupational Employment and Wage Statistics (OEWS). Hence, Lightcast is representative and maps well to actual hiring.[29] More importantly, the use of job postings rather than actual hiring removes possible confounding due to labor market frictions and search costs. The content of postings describes exactly who the firm is looking for, and is a clean measure of the firm's preferred composition of its human capital.

A second concern is the validity of using job search to test my model which treats investments in specific and general assets as irreversible investments. After all, workers can be fired or voluntarily quit. The short answer is that the model can easily accommodate partial reversibility.[30] The follow-up question is whether labor should be treated as fully reversible or partially reversible investments. In the labor economics and accounting literature, there is a large body of evidence suggesting that there are substantial, asymmetric frictions to adjust skilled labor.

In Appendix Figure IA.7, I compare hire, fire, and quit rates by major industry using data from the BLS. With the exclusion of jobs in the services industry, layoff and voluntary quit rates are quite low across major industries. At these magnitudes, they are comparable to capital depreciation rates used in the literature.[31] Moreover, they are less volatile than hiring rates, evidenced by their clustering below the 45° diagonal. This figure suggests that the firm can more easily adjust labor hiring, which is consistent with evidence in the literature.

In Appendix Figure IA.8, I show that employment growth in these industries coincides with investment growth in physical capital, using data from the Bureau of Economic Analysis. Appendix Table IA.14 is a formal regression. The coefficient in the regression of investment growth in fixed assets at t on employment growth at t is between 1.06 and 1.28, suggesting a good correspondence in aggregate hiring and investment decisions in the economy. Finally in Appendix Figure IA.9, I plot the average employment and employment growth by occupation code. The BLS does not keep track of hiring and firing by occupation category, but the plot provides evidence that volatility of total employment in "skilled" occupations is quite low, especially relative to the average growth rate of employment in these professions.

[29] Separately, I run a log-log regression of log # employees at $t+1$ on the log number of postings at t (and a constant) in my final sample of Lightcast firms matched to Compustat. A 1% increase in the number of postings predicts a 4% increase in the number of employees, though the within-firm adjusted R^2 in this regression is only 5%.

[30] Partial reversibility follows the solution method in Abel and Eberly 1996. Adapted to my model, the only changes are additional integration constants corresponding to the value of reallocating *outside* the firm, and additional free boundaries corresponding to the optimal time to sell assets.

[31] My model abstracted away from depreciation. But in continuous time depreciation rates are equivalent to Poisson events in which capital would break (akin to a worker stochastically leaving the firm), appearing through the discount rate. Hence, adjusting parameter values in δ^i, δ^j can take of this concern.

Taken together, I argue that studying investment in human capital through search is not too different from investment in physical capital. There is a good correspondence between the two, and there is supporting evidence that layoffs and quits are quantitatively small in practice.

3.4 Patent Data

The goal is to construct a measure analogous to Equation (20) that, for patent p, says

$$\begin{aligned}\text{General}_p = &\text{ average textual similarity to all other patents}\\ &\text{ outside of its own industry}\end{aligned} \tag{22}$$

Though patents do not directly share the same characteristics as labor, the specificity of a patent can proxy for the specificity of the human capital needed to produce these knowledge assets.

Constructing this measure can be divided into two steps: 1) compute similarity scores between patent p and every other patent, and 2) match all patents to an industry. The first step is computationally expensive, while the second step must identify patents, the ownership of which, have been transferred during acquisitions, and assign a single industry to patents filed by multi-segment firms. I discuss the procedure in detail in Appendix B.3 and provide a more formal definition of Equation 22.

Appendix Table IA.15 reports summary statistics on the General score of all matched patents in my sample, sorted by industry. Some of the most "general" industries with intense patent activity include:[32] Communications (48), Industrial Machinery and Computer Equipment (35), Electronic and Other Electric Equipment (36), Measuring, Medical and Optimal Equipment (38). Actually, the most "general" industry is Miscellaneous Retail (59), but this can be misleading since approximately 17,000 out of the 19,000 patents in this industry were filed by Amazon. The rank of Motion Pictures (78) is conceivable if one thinks of Netflix and many of the recent contenders in the market for streaming. Indeed, the high General score in this industry captures the idea that the human capital required to produce these patents likely have skills that can be put to work in other industries as well. In contrast, the most specific industries include: Agricultural Production - Crops (1), Tobacco Products (21), Apparel and Other Textile Products, Chemicals and Allied Products (28)

[32] One may be surprised by the presence of financial services towards the top of the list. Most of these patents are administrative in nature, such as inventory or personnel management. On the other hand for 2-digit SIC code 67, this industry includes the 4-digit code 6794: Patent owners & lessors so the high General comes from the patent owners having a diverse portfolio of patents. In contrast, patents from Security & Commodity Brokers (62) tend to be specific, and include patents for discounted cash flow analysis, derivative trading and so forth.

and Food and Kindred Products (20). Evidently, the knowledge capital required to produce patents for pesticides, cigarettes, diapers, drugs and beer have little outside use.

A firm's General score is the average score of its patents filed in year t. But I also include rolling averages over 3 and 5 year windows for robustness, requiring at most 1 year of missing data in the intermediate years for valid rolling averages. In Appendix Table IA.16, I show summary statistics for firms' scores in my final sample.

4 Empirical Analysis

Recall that the main testable predictions are the following:

- H1. Investment rates are increasing in firm-level risk
  - H1a. Multi-segment firms invest relatively less in their riskier segments, but this effect is offset the more unrelated the segment is to the firm's other businesses
- H2. Average specificity of investment is increasing in firm-level risk
  - H2a. However, this effect depends on the size of reallocation costs. If reallocation is not costly (or prohibitively costly), there are no cross-sectional differences in specificity as a function of uncertainty.

Prediction H2 is the main prediction while H1a and H2b are further conditional tests of the model.

4.1 Investment Rate

I run the following regression

$$\text{Investment rate}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1} \tag{23}$$

In a change of notation, going forward, I use subscript i to denote firm i and subscript j to denote the firm's primary industry. The dependent variable is the investment rate in physical capital I/K at $t+1$. The main independent variable of interest is $\text{Volatility}_{i,t}$ the firm's volatility at time t. Moreover, I include a vector of firm controls $\mathbf{x}_{i,t}$, firm fixed effects and industry $j \times$ year t fixed effects; this fixed effects specification removes unobserved time invariant differences across firms, and removes common effects that may exist for firms within a given industry-year. This, along with the control variables, helps isolate the cross-sectional relationship between uncertainty and the investment rate.

In Table 1, I show the results for the investment rate regressed on different measures of uncertainty. In the first three columns, I show that the results are robust to different fixed effects specifications and that the coefficients are economically similar in magnitude. Pervasively across different measures−the raw realized 5-year volatility from $t-5$ to $t-1$, the abnormal volatility that is stripped of any forcastable variation in expected returns, volatility on unlevered equity, monthly instead of daily and cash flow volatility over 20 quarters−the coefficient is robust, positive and economically significant. A roughly 5 percentage point increase in volatility is associated with a standard deviation increase in the investment rate. This result is also robust across different decade subsamples, though the most recent decade is only significant at the 10% level due to the covid year.

In Table 2, I use net hiring rates as the dependent variable and in Table 3, I use investment rates in knowledge capital as the dependent variable.[33] They share a common theme. Across physical, knowledge and human capital, higher firm-level risk is associated with higher investment.

Initially, it appears as though these stylized facts stand in contrast to the empirical literature that has predominantly found a negative relationship between uncertainty and investment. This is likely due to a difference in focus. Indeed, recent literature has focused on the time-series dimension, estimating the casual impact of uncertainty shocks starting with Bloom et al. 2007, Bloom 2009, Julio and Yook 2012, Gulen and Ion 2016, Kim and Kung 2016 and Alfaro et al. 2024; in these papers, the effect of uncertainty on physical capital investment is unambiguously negative. To my knowledge, the exception is Atanassov et al. 2024 who find causal evidence that political uncertainty increases intensity in research & development, which the authors argue is aligned with an exploratory and time-to-build view of R&D projects à la Bar-Ilan and Strange 1996. In my paper, which focuses on the cross-section, the evidence is mixed. While Leahy and Whited 1996, Gilchrist et al. 2014, Kermani and Ma 2023 document a negative coefficient in their physical capital regressions, Chang et al. 2024 find a positive one. The primary difference is that the former paper uses annual investment rate using volatility of equity measured over the previous year while the latter uses quarterly investment using volatility of *assets*. In this regard, my results support the evidence found in Chang et al. 2024, though the horizon at which uncertainty is measured is over 5 years instead of 1 year.

As Table 1 shows, the conflicting coefficients are not about the choice of capital. At least in the cross-section, the difference appears to be whether one focuses on equity versus asset

[33] As I discuss in Appendix B.2, I construct the stock of knowledge capital using the perpetual inventory method. Mechanically, there is a lag dependency in the dependent variable since the stock obeys a law of motion. Following Alfaro et al. 2024, I include the investment rate at t as a control to help correct for possible autocorrelation. As Angrist and Pischke 2009 caution however, there are concerns with estimation when one includes both fixed effects and lagged dependent variables. In Appendix Table IA.5, I drop firm fixed effects and show that the coefficients are slightly larger and the results appear to be much stronger statistically. As additional robustness, I study growth rates in R&D expenditures and intangible investments in Appendix Table IA.6−which avoids this problem since these are flows over flows instead of flows over stock−and find consistent results.

volatility, or whether the estimation window heavy relies on data from $t-1$ to t. In Figure 9, I re-run the regression specified in Equation (23) for different estimation window lengths and different measures of firm-level uncertainty. Each point corresponds to a separate regression and is the coefficient β_1 from that regression. The y-axis is the magnitude of the coefficient and the x-axis is the number of years used to estimate volatility.

Focusing on the solid blue line labeled "Vol", the coefficient on the one-year measure is -1.17. This is statistically significant at the 1% level and is consistent with findings in the literature. However as one lengthens the estimation window, the coefficients quickly turn positive. Even for the one-year measure, de-levering it by 1 plus the firm's gross debt-equity ratio reduces its magnitude, and it is no longer statistically distinguishable from zero.[34] The solid red line labeled "Lagged Vol" shows that these results are not due to the length of the window itself; when the one-year measure uses data from $t-2$ to $t-1$ instead of from $t-1$ to t, the coefficient is positive and also significant at the 1% level. Cross-sectionally, a negative coefficient appears to be the exception rather than the rule.

4.1.1 Discussion

The results from Figure 9 is a new fact and clearly shows that uncertainty measured from time $t-1$ to t contains different information. The one-year measure may correspond to "short-term" uncertainty while longer horizon measures, especially when de-levered, correspond to the average business risk that firms face, and is the appropriate measure for this paper supported by the theoretical results in Appendix C.3 with stochastic volatility. Simultaneously, this paper is the first to theoretically explain why changing the horizon at which uncertainty is measured can lead to a change in sign.

In my model, a positive relationship is explained by the effect of uncertainty on the firm's option to reallocate. Since a firm's price of risk matches its level of uncertainty, reallocation−which is optimally exercised several years down the line in expectation−is not very valuable for risky firms due to time discounting and a higher cost of capital. As a result, higher investment rates reflect the relative indifference to the composition of an investment project for risky firms who cannot afford to wait.

This explanation emphasizes the role of the timing of cash flows attached to an option payoff. It also makes it possible to reconcile the contrasting signs at different horizons if

[34] All the coefficients are statistically significant at the 5% level except for following: the one-year for "Vol (de-levered)" and the two-year and three-year for "Vol" and "Abnormal Vol". These five points are not distinguishable from zero.

Table 1: This table is the panel regression:

$$\text{Investment rate}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

The dependent variable is the investment rate I/K at $t+1$, calculated as capex divided by the average of then current and lagged net PP&E following Belo et al. 2014. The right-hand side includes a vector of firm controls $\mathbf{x}_{i,t}$, firm i and industry j × year t fixed effects, and the main variable of interest: firm volatility at t. Volatility is computed in several ways: Vol_{5y} is the annualized volatility of daily excess return stocks from June $t-5$ to June $t-1$. Abnormal Vol_{5y} is the volatility of residuals from a regression of firm i excess stock returns on the Fama-French five-factor model. Vol_{5y} (de-levered) is Vol_{5y} divided by 1 plus the firm's gross debt-equity ratio. Vol_{5y} (monthly) uses monthly instead of daily data. Within a calendar year, stock returns are required to have a minimum of 230 observations. Finally, Cashflow Vol_{5y} is the volatility of 5 year quarterly sales from Compustat quarterly data (20 obs), normalized by lagged assets. Firm controls include: Tobin's Q, the Whited-Wu index, payout, tangibility (Leary and Roberts 2014), book leverage, operating cash flows, market-to-book, log sales, log assets, intangible investment (Peters and Taylor 2017), investment rate at t, return on assets, r&d/assets, capex/assets and sales growth. Moreover, following Alfaro et al. 2024, I include the compounded excess stock returns over the 5 year period as a first-moment control when incorporating return based measures of volatility. Data variables are winsorized at the 1st and 99th percentiles, and I require net PP&E to be greater than 5 million as is standard in the literature. Coefficients are multiplied ×100.

Dependent Variable: Physical Capital Investment $\text{rate}_{i,t+1}$

	1.	2.	3.	4.	5.	6.	7.
Vol_{5y}	1.03**	1.44***	1.42***				
	(2.50)	(2.99)	(2.77)				
Abnormal Vol_{5y}				1.72***			
				(3.43)			
Vol_{5y} (de-levered)					2.36***		
					(3.77)		
Vol_{5y} (monthly)						1.38***	
						(3.07)	
Cashflow Vol_{5y}							3.60***
							(4.10)
Firm controls	Yes						
Fixed effects	Firm, Year	Firm, ff-49 × Year	Firm, sic-2 × Year	Firm, sic-2 × Year			
Standard errors	Firm, Year	ff-49, Year	sic-2, Year	sic-2, Year			
Adjusted R^2	0.56	0.57	0.57	0.57	0.57	0.57	0.58
Observations	68,782	68,782	68,782	68,782	68,782	68,782	71,079

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table 2: This table is the panel regression:

$$\text{Net Hiring Rate}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

The dependent variable is the net hiring rate at $t+1$, calculated as $(\text{emp}_{t+1} - \text{emp}_t)/(.5\text{emp}_{t+1} + .5\text{emp}_{t+1})$ where emp is the Compustat variable for employment. The right-hand side includes a vector of firm controls $\mathbf{x}_{i,t}$, firm i and industry j × year t fixed effects, and the main variable of interest: firm volatility at t. Volatility is computed in several ways: Volatility is computed in several ways: Vol_{5y} is the annualized volatility of daily excess return stocks from June $t-5$ to June $t-1$. Abnormal Vol_{5y} is the volatility of residuals from a regression of firm i excess stock returns on the Fama-French five-factor model. Vol_{5y} (de-levered) is Vol_{5y} divided by 1 plus the firm's gross debt-equity ratio. Vol_{5y} (monthly) uses monthly instead of daily data. Within a calendar year, stock returns are required to have a minimum of 230 observations. Finally, Cashflow Vol_{5y} is the volatility of 5 year quarterly sales from Compustat quarterly data (20 obs), normalized by lagged assets. Firm controls include: Tobin's Q, the Whited-Wu index, payout, tangibility (Leary and Roberts 2014), book leverage, operating cash flows, market-to-book, log sales, log assets, intangible investment (Peters and Taylor 2017), investment rate at t, return on assets, r&d/assets, capex/assets and sales growth. Moreover, following Alfaro et al. 2024, I include the compounded excess stock returns over the 5 year period as a first-moment control when incorporating return based measures of volatility. Data variables are winsorized at the 1st and 99th percentiles. Coefficients are multiplied ×100.

Dependent Variable: Net Hiring $\text{Rate}_{i,t+1}$

	1.	2.	3.	4.	5.	6.	7.
Vol_{5y}	2.37^{***}	2.73^{***}	2.24^{**}				
	(2.72)	(2.80)	(2.41)				
Abnormal Vol_{5y}				2.57^{**}			
				(2.57)			
Vol_{5y} (de-levered)					2.65^{***}		
					(2.82)		
Vol_{5y} (monthly)						2.50^{***}	
						(3.20)	
Cashflow Vol_{5y}							4.28^{***}
							(2.69)
Firm controls	Yes						
Fixed effects	Firm, Year	Firm, ff-49 × Year	Firm, sic-2 × Year	Firm, sic-2 × Year			
Standard errors	Firm, Year	ff-49, Year	sic-2, Year	sic-2, Year			
Adjusted R^2	0.16	0.18	0.18	0.18	0.18	0.18	0.18
Observations	80,864	80,864	80,864	80,864	80,864	80,864	83,315

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table 3: This table is the panel regression:

$$\text{R\&D Investment rate}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

The dependent variable is the R&D investment rate at $t+1$, calculated as R&D expenditures divided by the average of then current and lagged stock of *knowledge capital*. I construct the stock of knowledge capital using the perpetual inventory method from past R&D expenses as detailed in Appendix B.2. The right-hand side of this regression includes a vector of firm controls $\mathbf{x}_{i,t}$, firm i and industry j $\times$ year t fixed effects, and the main variable of interest: firm volatility at t. Construction of the volatility measures are identical as before. Firm controls include: Tobin's Q, lagged R&D Investment rate, the Whited-Wu index, payout, tangibility (Leary and Roberts 2014), book leverage, operating cash flows, market-to-book, log sales, log assets, intangible investment (Peters and Taylor 2017), return on assets, r&d/assets, capex/assets and sales growth. Moreover, following Alfaro et al. 2024, I include the compounded excess stock returns over the 5 year period as a first-moment control when incorporating return based measures of volatility. Data variables are winsorized at the 1st and 99th percentiles. Coefficients are multiplied $\times$100.

Dependent Variable: R&D Investment rate$_{i,t+1}$

	1.	2.	3.	4.	5.	6.	7.
Vol_{5y}	1.14***	1.31**	1.37***				
	(3.24)	(2.50)	(2.74)				
Abnormal Vol_{5y}				1.61***			
				(3.50)			
Vol_{5y} (de-levered)					2.08***		
					(3.62)		
Vol_{5y} (monthly)						1.00***	
						(3.02)	
Cashflow Vol_{5y}							1.58***
							(3.25)
Firm controls	Yes						
Fixed effects	Firm, Year	Firm, ff-49 × Year	Firm, sic-2 × Year	Firm, sic-2 × Year			
Standard errors	Firm, Year	ff-49, Year	sic-2, Year	sic-2, Year			
Adjusted R^2	0.83	0.84	0.84	0.84	0.84	0.84	0.84
Observations	39,987	39,987	39,987	39,987	39,987	39,987	40,628

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

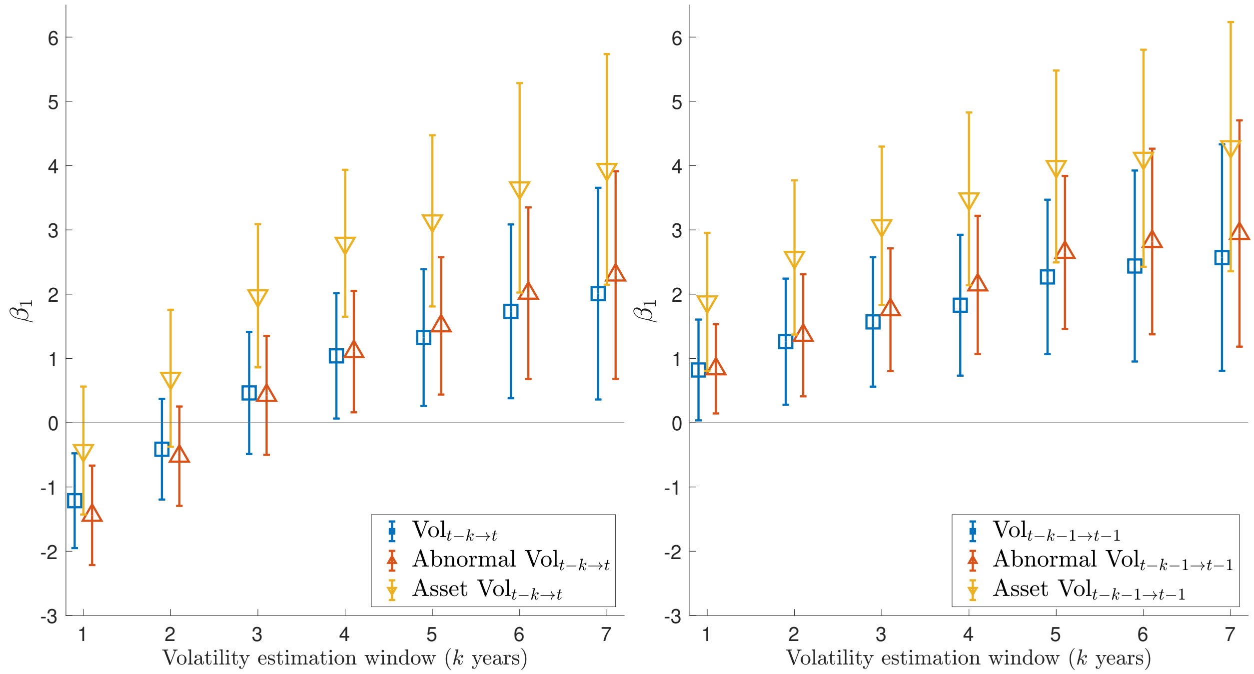


Figure 9: **Coefficients from investment regressions for different volatility estimation windows.**. This figure plots the coefficient β_1 from the panel regression:

$$\text{Investment rate}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

using different estimation windows for firm i $\text{Volatility}_{i,t}$. The dependent variable is the investment rate I/K at $t+1$ as described in Table 1. The x-axis corresponds to the length of the estimation window, in years, used to compute realized volatility. "Realized Vol" is the annualized realized volatility from June $t-k$ to June t, while "Lagged Realized Vol" is from June $t-k-1$ to June $t-1$. When estimation includes information up to t, the denominator in the investment rate calculation introduces up to half a year of simultaneity. "Abnormal Vol" is the volatility of residuals from Fama-French regressions, and "Asset Vol" is volatility de-levered by 1 plus the gross debt-equity ratio.

volatility is stochastic. When short bursts of high uncertainty are followed by prolonged periods of low uncertainty, an uncertainty shock today implies that reallocation is expected to occur in a low volatility environment. As a result, the present value of flexibility is higher when the firm faces a period of high uncertainty because the expected discount rate used to price the distant benefits of reallocation is lower. Consequently, the firm is more sensitive to project composition and is more willing to delay or reduce investment during high uncertainty regimes. This reconciles why high short-term uncertainty can lead to a negative sign in contrast to its behavior on average. To my knowledge, this is the first paper that documents and can explain the change in sign as a function of the length at which uncertainty is measured.

4.2 Specificity

I now examine hypothesis H2: specificity is increasing in firm-level uncertainty. If volatile firms invest more due to their relative indifference towards project composition, we should observe differences in the specificity of their capital investments.

Human Capital

In Table 4, I show results from the panel regression for firm i in primary industry j at t

$$\text{General}_{i,t+k} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

for $k \in \{1, 2\}$ years. Since the dependent variable measures the extent the which a firm desires workers with general skills, higher specificity corresponds to a negative coefficient for β_1.

Consistent with the theoretical predictions, firms with higher uncertainty are associated with higher specificity. In economic magnitudes, a 5 percentage point increase in volatility is associated with a 1.25 standard deviation increase in worker specificity. The sign of the coefficient raises an interesting observation. Firms that would most benefit from actively planning to smooth output across adverse states choose not to do so.

In Appendix Table IA.8, I show that the coefficients are robust to different fixed effects specifications. In fact, the coefficients hardly change at all. In Appendix Table IA.9, I repeat the regression, but only keep firm-level risk and fixed effects as controls to check whether these results are due to over-fitting or the inclusion of a particular control variable. For two of the three measures of uncertainty, the coefficients become stronger both economically and statistically. The exception is the measure for de-levered volatility; when one does not include the firm's leverage ratio as a control, the economic magnitude on the de-levered volatility measure drops considerably.

In Appendix Table IA.10, I show the control variables that have economically large coefficients and/or are statistically significant. Moreover, I include different constructions of the General measure. Ex-ante, one concern is the extent to which the measure corresponds to the scale of the firm. The stability of the point estimates depending on whether one includes or excludes back and middle office positions such as finance, human resources and compliance suggests that this is not the case. Overall, higher sales, Tobin's q and firm size at t are associated with a desire for more general workers at $t + 1$, while higher leverage and volatility are associated with a desire for more specific workers. In terms of point estimates for well-estimated variables, volatility has the largest magnitude followed by sales.

Having established the robustness in the relationship between firm-level uncertainty and specificity, I study whether the motive to reallocate generalists is a plausible mechanism

Table 4: This table is the panel regression:

$$\text{General}_{i,t+k} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

for $k \in \{1,2\}$ years. The dependent variable $\text{General}_{i,t+k}$ measures how similar the desired skills in firm i's time $t+k$ job vacancy postings are to the specialized skill vectors in all industries outside of firm i's own. It is the average cosine similarity over firm i and each outside industry's vector of skills using the term-frequency-inverse-document-frequency (tf-idf) approach. Further details are in Section 3.3.2. The independent variables include a vector of firm controls $\mathbf{x}_{i,t}$, firm i and industry $j \times$ year t fixed effects, and the main variable of interest: Volatility is computed in several ways: Vol_{5y} is the annualized volatility of daily excess return stocks from June $t-5$ to June $t-1$. Abnormal Vol_{5y} is the volatility of residuals from a regression of firm i excess stock returns on the Fama-French five-factor model. Vol_{5y} (de-levered) is Vol_{5y} divided by 1 plus the firm's gross debt-equity ratio. Within a calendar year, stock returns are required to have a minimum of 230 observations. Firm controls include: Tobin's Q, the Whited-Wu index, log sales, payout, tangibility (Leary and Roberts 2014), return on assets, book leverage, log assets, intangible investment at t (Peters and Taylor 2017), investment rate I/K at t, r&d/assets, capex/assets, firm and primary industry sales growth. Moreover, following Alfaro et al. 2024, I include the compounded excess stock returns over the 5 year period as a first-moment control when incorporating return based measures of volatility. Data variables are winsorized at the 1st and 99th percentiles. Coefficients are multiplied $\times 100$. Data is annual, $2010-2023$.

Dependent Variable: $\text{General}_{i,t+k}$

	$\text{General}_{i,t+1}$			$\text{General}_{i,t+2}$		
	1.	2.	3.	4.	5.	6.
Vol_{5y}	-0.27**			-0.29***		
	(-2.40)			(-3.18)		
Abnormal Vol_{5y}		-0.35**			-0.37***	
		(-2.78)			(-3.77)	
Vol_{5y} (de-levered)			-0.33**			-0.45***
			(-2.29)			(-3.48)
Firm controls		Yes			Yes	
Fixed effects		Firm, sic-2 $\times$ Year			Firm, sic-2 $\times$ Year	
Standard errors		sic-2, Year			sic-2, Year	
Adjusted R^2	0.70	0.70	0.70	0.70	0.70	0.70
Observations	16,711	16,711	16,711	14,256	14,256	14,256

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

behind these results. To do this, I first calculate the average distance between all job positions extracted from a firm's job postings in a calendar year. The idea is the following. If the firm's workers are all based in San Francisco, the costs of reallocating workers to help produce a different good is likely to be lower than if workers were split between San Francisco and New York City; aside from the relocation expense itself, there are likely to be fewer impediments to transfer workers across different units when say, everyone works in the same building.

Using the average distance to proxy for the potential reallocation costs lends itself to a precise test of the mechanism in my model. For a firm whose businesses are very volatile, *because* reallocation is costly, its expected benefits are too far into the future to be important for planning decisions today; the firm's price of risk discounts faraway cashflows at too high of a rate. It follows that any change in the expected waiting period affects the extent to which cross-sectional differences between firm-level risk and average investment specificity can be observed. If the firm can costlessly move workers back and forth, then the benefits of reallocation would occur much sooner, muddying the relationship between the desired specificity of workers and volatility.[35]

Formally, I run the regression:

$$\begin{aligned}\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \beta_2 \text{Avg Distance}_{i,t} \\ + \beta_3 \text{Volatility}_{i,t} \times \text{Avg Distance}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}\end{aligned}$$

The coefficient of interest is β_3, the interaction between volatility and average distance. If the average distance between all job positions does indeed capture an essence of adjustment costs, the combined effect β_3 shows how the link between volatility and specificity can magnify or subdue based on how spread out the firm's workers are. Table 5 shows the results. As columns (2), (4) and (6) show, the coefficient β_3 is negative. Because reallocation is costly, firms avoid specific assets because they cannot afford to delay reallocation. But if reallocation is costly and the firm is risky, the firm loses the need to avoid specific assets.

In Appendix Table IA.11, instead of the average distance, I use a dummy variable that takes the value of 1 if the average distance is greater than zero (the firm is hiring in more than one metropolitan statistical area); I find consistent results. In Appendix Table IA.12, I split the sample based on whether the average distance of job positions is above or below the mean for that year; the coefficient on volatility is statistically significant for firms above the mean distance but not below.

To verify whether the distance variable captures an element of adjustment costs rather than firms that have a nation-wide presence, I run placebo tests and interact firm volatility with sales, firm size and firm scope from Hoberg and Phillips 2024. Appendix Table IA.13

[35] From the theory, cross-sectional differences in average specificity are not detectable across different values of firm-level volatility when the adjustment cost c either $c \to 0$ or $c \to \infty$.

Table 5: This table is the panel regression:

$$\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \beta_2 \text{Avg Distance}_{i,t} + \beta_3 \text{Volatility}_{i,t} \times \text{Avg Distance}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

I repeat the analysis from Table 4 but add an interaction term for the average distance between the coordinate pairs in which any two job positions are based, across all possible combinations using location data extracted from firm i's job postings at t. For example, if firm i posted 1,000 postings at t, then there are $1000 \times 999/2 \approx 500,000$ combinations. The variable "Avg Distance" is log 1+average distance, where average distance is the average distance across these 500,000 combinations. When the number of combinations exceed 10 million, I randomly sample (with replacement) 10 million pairs.

Dependent Variable: $\text{General}_{i,t+1}$

	1.	2.	3.	4.	5.	6.
Avg Distance	0.04***	0.08***	0.04***	0.09***	0.04***	0.06***
	(3.39)	(5.92)	(3.38)	(7.59)	(3.34)	(4.18)
Vol_{5y}	-0.27**	0.27				
	(-2.36)	(1.57)				
Avg Distance × Vol_{5y}		-0.08***				
		(-3.52)				
Abnormal Vol_{5y}			-0.35**	0.35*		
			(-2.71)	(1.97)		
Avg Distance × Abnormal Vol_{5y}				-0.11***		
				(-4.62)		
Vol_{5y} (de-levered)					-0.32**	0.11
					(-2.23)	(0.48)
Avg Distance × Vol_{5y} (de-levered)						-0.07*
						(-2.06)
Firm controls	Yes					
Fixed effects	Firm, sic-2 × Year					
Standard errors	sic-2, Year					
Adjusted R^2	0.70	0.70	0.70	0.70	0.70	0.70
Observations	16,711	16,711	16,711	16,711	16,711	16,711

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

shows the results. There are zero effects when interacting with these variables. On the other hand, the results are robust when interacting with a distance measure that excludes job positions in sales & customer support; excluding these positions show that distance is not simply capturing retailers who operate across the continental U.S..

Knowledge Capital

In Table 6, I repeat the regression with the general measure constructed using patent data. This has the benefit that the date is from 1976 to patents granted by 2022, but because not all firms actively patent, the increase in sample size is not too large. Instead, this patent-based measure is intended to proxy for the assets inside a company (including workers) necessary to produce these outputs, and supplements the vacancy posting based measure. As there can be a margin of error classifying patents filed by multi-segment firms (described in B.3.3), I focus on single-segment industry firms in columns (4)-(6). Moreover, focusing on single-segment firms removes the confounding effect on the endogenous choice to diversify and firm specificity.

Consistent with model predictions, high firm-level uncertainty is associated with higher specificity. The coefficients are very similar to the coefficients in the regressions with the job postings data, showing a robust counter-intuitive relationship between firm-level uncertainty and investment specificity.

Physical Capital

I narrow the scope of analysis to multi-segment firms. The empirical literature on conglomerates often stresses the endogenous choice to diversify, and the selection of which industry the firm diversifies to. This is especially problematic if the choice to operate in a different industry is to minimize volatility and create value through the discount rate channel, for example, as in Lewellen 1971 and Hann et al. 2013. Given the possible confounding effect on the firm's volatility, I avoid mixing single-segment and multi-segment firms in the analysis, set by Fama-French 49 industry group classifications[36]. Though in principle, my theory also applies to single-industry firms who have different businesses (for example, NVIDIA Corp's consumer PC's and AI solutions units), focusing on the latter set of firms is likely to mitigate the effect of organizational choice when all firms in my sample are exposed to the common problem.

I use industry-level measures of redeployability from Kim and Kung 2016. To summarize their measure, suppose a large fraction of an industry's capital expenditures is on a machine that is also purchased by many other industries. Then this industry's redeployability

[36] As a precedent, Boguth et al. 2022 also use Fama-French industry groups to identify the set of conglomerates in their sample and restrict their analysis to multi-segment firms

Table 6: This table is the panel regression;

$$\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_t + \varepsilon_{i,t+1}$$

The dependent variable is the average textual similarity of patents that firm i files at date $t+1$, relative to all patents filed by firms outside of firm i's own industry. The independent variables include a vector of firm controls $\mathbf{x}_{i,t}$, firm i and industry j × year t fixed effects, and the main variable of interest: firm volatility at t. Vol_{5y} is the annualized volatility on the firm's excess stock returns using daily data from $t-5$ to t as of June for that fiscal year, then lagged by one year to avoid possible concerns with reverse causality. Following Kermani and Ma 2023, Abnormal Vol_{5y} is the volatility of residuals from a regression of firm i excess stock returns on the Fama-French five-factor model, also lagged by one year. Vol_{5y} (de-levered) is Vol_{5y} divided by 1 plus the firm's gross debt-equity ratio. Within a calendar year, stock returns are required to have a minimum of 230 observations. Firm controls include: knowledge capital at t (from a previous table, constructed using the perpetual inventory method on past R&D expenditures), Tobin's Q, the Whited-Wu index, log sales, payout, tangibility (Leary and Roberts 2014), return on assets, book leverage, log assets, intangible investment at t (Peters and Taylor 2017), investment rate I/K at t, r&d/assets, capex/assets, firm and primary industry sales growth. Moreover, following Alfaro et al. 2024, I include the compounded excess stock returns over the 5 year period as a first-moment control when incorporating return based measures of volatility. Data variables are winsorized at the 1st and 99th percentiles. Coefficients are multiplied ×100. Data is annual, $1976-2020$.

Dependent Variable: General via $\text{Patents}_{i,t+1}$

	Full Sample			Single Sector Firms Only		
	1.	2.	3.	4.	5.	6.
Vol_{5y}	-0.23***			-0.31**		
	(-4.50)			(-3.70)		
Abnormal Vol_{5y}		-0.21***			-0.26**	
		(-3.78)			(-3.44)	
Vol_{5y} (de-levered)			-0.20*			-0.22
			(-2.14)			(-1.76)
Firm controls		Yes			Yes	
Fixed effects		Firm, sic-2 × Year			Firm, sic-2 × Year	
Standard errors		sic-2, Year			sic-2, Year	
Adjusted R^2	0.56	0.56	0.56	0.64	0.64	0.64
Observations	22,675	22,675	22,675	13,057	13,057	13,057

t statistics in parentheses

* $p < 0.05$,

** $p < 0.01$

*** $p < 0.001$

score is high, suggesting that investments are relatively reversible because secondary markets for these same production inputs likely exist. In contrast, a low redeployability score suggests that an industry's main production inputs are not used by others and therefore less reversible, coinciding with the characteristics of specific assets in my model.[37] Because the redeployability score is the weighted by the firm's identifiable assets in each of its businesses, changes to the score can come from increased investment to one business, and/or reallocation of assets from one business to another.

In Table 7, I repeat the regression. The negative coefficient suggests that higher volatility is associated with a shift towards *less* redeployable divisions, up to 3 years in the future. The measure of volatility as described in the table caption uses weighted-average industry volatilities via Fama-French portfolios, which helps account for cross-industry correlations by multi-segment firms and removes any concerns of reverse causality. In economic magnitudes, a one percentage point increase in volatility is associated with a one-third standard deviation decrease in redeployability. For physical capital as well, specificity is increasing in firm-level risk.

4.3 Within-Firm Analysis

Now, I turn to the within-firm prediction H1a. In Proposition 4, I showed that when a multi-segment firm pairs together two businesses that differ considerably in their level of risk, the firm restrains investment in the riskier business because of the high value of offloading production to a business with a lower price of risk. However, as Corollary 4 and Figure 5 showed, this effect is decreasing in the adjustment cost of reallocation; it must first be feasible to reallocate assets for this effect to take hold. In Section 3.3.2, I discussed a potential proxy for the adjustment cost of transforming general assets within a multi-segment firm by using the relatedness between a firm's segments, measured by the overlap of worker skills.

Prediction H1a is a refinement of the theory and makes the following claim. If a firm operates in multiple sectors that use a common set of production inputs but one business is much riskier than the others, all else equal, it does not make much sense to invest as intensively in the riskier business; the firm cannot avoid investments in specific, dedicated technology to expand its footprint in a business, but specific investments impose a heavier cost when volatility is asymmetric. On the other hand if the firm's businesses are completely disjoint, then the firm should treat each as separate units rather than as if it were piecing

[37] As a caveat, their measure uses the 1997 BEA capital flows table to estimate redeployability based on industries' expenditures on a physical asset category. However, time-series variations in their redeployability score comes from variations in the total market value of industries rather than changes to an industry's expenditures to different assets. As a result, there is concern on the internal validity of holding an industry's expenditures on a particular asset category, which is why I do not extend their method past their original sample from 1985-2015. Moreover, it is possible that changes in the redeployability score may be correlated with changes to volatility, confounding the point estimates in this regression.

Table 7: **Changes in physical capital specificity via industry segments.** This table is the panel regression:

$$\text{Redeployability}_{i,t+k} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \cdot \text{Volatility}_{i,t} + \delta_i + \gamma_t + \varepsilon_{i,t+k}$$

The dependent variable is the redeployability score from Kim and Kung 2016. The redeployability score is measured at the industry level, so firm i's score at date $t+k$ for $k = \{1, 2, 3\}$ years is the weighted average of its industry scores. The right-hand side includes a vector of firm controls $\mathbf{x}_{i,t}$, firm i and year t fixed effects, and the main variable of interest: firm volatility at t. Data is annual, from 1985-2015. The sample is restricted to the set of firms that report more than 1 Fama-French 49 industry group in Compustat segments at fiscal years t and $t+k$. Firm volatility at t is $V = (w^\top \text{diag}(s) \cdot \Omega \cdot \text{diag}(s)w)^{1/2}$, where w is a vector of a firm's industry weights in Compustat segments at date t via its identifiable assets (iat), s is a vector of industry volatilities as of June in year t de-levered by the industry's gross D/E ratio, and Ω is a correlation matrix of industry excess returns. Volatilities and correlations are estimated via daily data over 5-year rolling windows using Fama-French 49 industry portfolio returns. Firm controls include: tangibility of assets, Tobin's Q, the Whited-Wu index, return on assets, book leverage, log assets, log sales, payout, intangible investment, investment rate, r&d/assets, ebit/assets, capex/assets, firm and primary industry sales growth.

Dependent Variable: Redeployability Score$^i_{t+k}$

	Horizon k years			Horizon k years		
	+1	+2	+3	+1	+2	+3
Volatility$_{i,t}$	-0.14***	-0.08***	-0.06***	-0.14***	-0.08***	-0.06***
	(-15.73)	(-8.97)	(-5.64)	(-10.36)	(-7.04)	(-4.31)
Firm controls		Yes			Yes	
Fixed effects		Firm, Year			Firm, sic-2 × Year	
Standard errors		Firm, Year			sic-2, Year	
Adjusted R^2	0.85	0.84	0.84	0.85	0.84	0.84
Observations	18,310	17,251	16,319	18,310	17,251	16,319

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

together parts of a whole.

To test this prediction, I run the following regression,

$$\begin{aligned}\text{Segment Net Hiring rate}_{i,j,t+1} = \mathbf{x}_{j,t}\boldsymbol{\xi} + \beta_1\text{Vol}_{j,t} + \beta_2\text{Relatedness}_{i,j} \\ + \beta_3\text{Vol}_{j,t} \times \text{Relatedness}_{i,j} + \gamma_{i,t} + \varepsilon_{i,j,t+1}\end{aligned}$$

Since the proxy for adjustment costs is a labor-based measure, the dependent variable is firm i segment j net hiring rate, imputed using the firm's net hiring rate scaled by the segment's sales weight. The Firm × Year fixed-effects specification isolates variation to the within-firm level. The main coefficient of interest is β_3 in the above equation, the interaction between segment j volatility and the average relatedness between segment j and all other segments for firm i at t.

In Table 8 column (1), the coefficient β_1, though positive, is not statistically different from zero. This supports the prediction that if the firm pairs together businesses that are completely unrelated, then there should be no observable differences in their relative investment rates despite the volatility of the business itself.

However, the more related one segment is to the rest of the firm, the more careful the firm is about project selection the riskier this segment is. The coefficient β_3 is negative as predicted.

In column (2), I repeat the regression but calculate the relatedness between two industries using the full sample of job postings from 2010-2023 instead of calculating industry-relatedness each year; here, relatedness between two industries becomes a time-invariant characteristic. The coefficients hardly change. In columns (3)-(5), I assume industry-relatedness is a fixed characteristic and assume it holds more generally across different time periods. The point-estimates for β_3 monotonically declines as one extends the data sample, but is statistically significant at the 1% level for 2000-2023, while significant at the 10% level for 1990-2023.

In Table 9, I run the identical regression using patent-based measures of relatedness and find similar economic magnitudes; this should be of no surprise. Labor and patent based measures of relatedness show good correspondence in industries where data on the latter is available as shown in Appendix Figure IA.10.

These regressions show that riskiness itself does not explain differences in investment rates at the segment level. Differences in risk must be accompanied by the ability to share production inputs for these differences to affect capital planning.

Table 8: This table is the panel regression:

$$\text{Segment Net Hiring rate}_{i,j,t+1} = \mathbf{x}_{j,t}\boldsymbol{\xi} + \beta_1 \text{Vol}_{j,t} + \beta_2 \text{Relatedness}_{i,j} + \beta_3 \text{Vol}_{j,t} \times \text{Relatedness}_{i,j} + \gamma_{i,t} + \varepsilon_{i,j,t+1}$$

This regression narrows the scope of analysis to firms that report more than one industry segment in Compustat segments, defined at the 2-digit SIC level. The dependent variable is firm i's net hiring rate in segment j at time $t+1$, imputed by the firm's net employee growth scaled by segment j's weight as a fraction of total sales. The main coefficient of interest is β_3 in the above equation, the interaction between segment j volatility and the *Relatedness* score between segment j and all other segments for firm i at t, $\mathcal{G}_{i,t} \setminus \{j\}$ where $\mathcal{G}_{i,t}$ denotes the set of reported industry segments for firm i at t. Volatility at the segment level is calculated as the median industry Vol_{5y} at t. The relatedness score is the cosine similarity between industry skill vectors from job postings data. A high score suggests that there is strong overlap between the recommended skills in business j and the firm's other businesses. $\text{Relatedness}_{i,j}$ (fixed) is a time-invariant characteristic that uses the full sample estimates while the former is calculated each year. The right-hand side of this regression includes a vector of segment and industry controls $\mathbf{x}_{j,t}$. Segment controls include firm i segment j's log sales, log assets, sales growth and segment weight at t. Industry j controls use median values for firms whose primary industry is in that industry at t, and include: Tobin's Q, Investment rate (I/K), the Whited-Wu index, log sales, payout, tangibility (Leary and Roberts 2014), return on assets, book leverage, log assets, intangible investment (Peters and Taylor 2017), capex/assets, and sales growth. Moreover, following Alfaro et al. 2024, I include the industry average compounded excess stock returns over the preceding 5 year period as a first-moment control when incorporating return based measures of volatility. Coefficients are multiplied $\times 100$.

Dependent Variable: Segment Net Hiring $\text{rate}_{i,j,t+1}$

	1.	2.	3.	4.	5.
$\text{Vol}_{j,5y}$	5.78	5.37	3.68*	4.54	4.42*
	(1.10)	(1.24)	(1.91)	(1.51)	(1.78)
$\text{Relatedness}_{i,j}$	1.98***				
	(3.32)				
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$	-3.45**				
	(-2.59)				
$\text{Relatedness}_{i,j}$ (fixed)		1.95**	1.01**	0.90*	0.77**
		(2.88)	(2.82)	(1.82)	(2.17)
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$ (fixed)		-3.49**	-1.98***	-1.78*	-1.53**
		(-2.47)	(-3.23)	(-1.96)	(-2.14)
Start Year	2010	2010	2000	1990	1976
End Year	2023	2023	2023	2023	2023
Controls			Segment, Industry		
Fixed effects			Firm×Year		
Standard errors			Firm, Year		
Adjusted R^2	0.54	0.54	0.47	0.63	0.62
Observations	8,546	8,546	17,893	29,157	46,067

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table 9: This table is the panel regression:

$$\text{Segment Net Hiring rate}_{i,j,t+1} = \mathbf{x}_{j,t}\boldsymbol{\xi} + \beta_1 \text{Vol}_{j,t} + \beta_2 \text{Relatedness}_{i,j} + \beta_3 \text{Vol}_{j,t} \times \text{Relatedness}_{i,j} + \gamma_{i,t} + \varepsilon_{i,j,t+1}$$

I repeat the analysis from Table 8 but using measures of industry relatedness using patent data. $\text{Relatedness}_{i,j}$ is the average textual similarity of patents in industry j to all other industries to which firm i at t operates. $\text{Relatedness}_{i,j}$ (doc) uses the fraction of patents that exceed 5% similarity instead of the textual similarity score itself. $\text{Relatedness}_{i,j}$ (single) uses patents filed by only single-segment firms. $\text{Relatedness}_{i,j}$ (raw) removes scaling effects due to the fact that patents in different industries differ in how self-similar they are to patents filed in their own industries.

Dependent Variable: Segment Net Hiring $\text{rate}_{i,j,t+1}$

	1.	2.	3.	4.	5.	6.
$\text{Vol}_{j,5y}$	1.31	2.25**	1.20	2.33**	2.53**	2.92**
	(1.43)	(2.02)	(1.38)	(2.16)	(2.48)	(2.32)
$\text{Relatedness}_{i,j}$	3.75*					
	(1.86)					
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$	-8.41**					
	(-2.42)					
$\text{Relatedness}_{i,j}$ (doc)		3.94**				
		(2.21)				
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$ (doc)		-8.82***				
		(-2.77)				
$\text{Relatedness}_{i,j}$ (single only)			3.03*			
			(1.88)			
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$ (single only)			-5.91**			
			(-2.15)			
$\text{Relatedness}_{i,j}$ (doc, single only)				3.76**		
				(2.45)		
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$ (doc, single only)				-7.37***		
				(-2.71)		
$\text{Relatedness}_{i,j}$ (raw)					0.45**	
					(2.47)	
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$ (raw)					-0.93***	
					(-3.16)	
$\text{Relatedness}_{i,j}$ (doc, raw)						0.50**
						(2.24)
$\text{Vol}_{j,5y} \times \text{Relatedness}_{i,j}$ (doc, raw)						-1.04***
						(-2.73)
Firm controls	Yes					
Fixed effects	Firm, sic-2 × Year					
Standard errors	sic-2, Year					
Adjusted R^2	0.57	0.57	0.57	0.57	0.57	0.57
Observations	38,655	38,655	38,655	38,655	38,655	38,655

t statistics in parentheses
* $p < 0.10$
** $p < 0.05$
*** $p < 0.01$

5 Conclusion

This article develops a dynamic model of reallocation and investment. Principally, I introduce a distinction between general, multipurpose assets that can be moved between product markets, and specific, dedicated assets that cannot be moved. I spell out a new mechanism. Flexibility creates value but investing in specific assets reduces it. The firm accounts for this tension and substantially delays specific investments in the perpetual option case, and outright skips them in the multiple arrival case.

If the purpose of collecting multipurpose assets is to use them somewhere else, one would suspect that the incentive to collect multipurpose assets is greater when volatility is high. I show the exact opposite. Because discount rates are risk-adjusted, distant cash flows become less valuable to the firm. Since reallocation does not happen any sooner, investment decisions today no longer depend on decisions that are further out into the future. As a result, the firm adopts more projects on average when volatility is high compared to when volatility is low, irrespective of the composition of the investment project. These findings show how the firm can tailor one action entirely to preserve another potential action.

Consistent with the main mechanism outlined in this paper, I document a positive association between medium to long-run uncertainty and investment. This paper argues that one must appeal to the composition of investment projects to understand this positive relationship.

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A Appendix: Proofs

A.1 Proof of Proposition 1

Solving under the risk-neutral measure for the firm's value function $J(t, A^i, A^j, k^i, k^j)$, in the region of inaction, the HJB equation associated with the firm's problem is

$$
\begin{aligned}
rJ(A^i, A^j, k^i, k^j) = & \left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} A^i + \left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} A^j \\
& + A^i(\mu^i - \kappa^{(1,0)}\sigma^i)\frac{\partial J(A^i, A^j, k^i, k^j)}{\partial A^i} + \frac{1}{2}(A^i\sigma^i)^2\frac{\partial^2 J(A^i, A^j, k^i, k^j)}{\partial A^{i2}} \\
& + A^j(\mu^j - \kappa^{(0,1)}\sigma^j)\frac{\partial J(A^i, A^j, k^i, k^j)}{\partial A^j} + \frac{1}{2}(A^j\sigma^j)^2\frac{\partial^2 J(A^i, A^j, k^i, k^j)}{\partial A^{j2}} \\
& + A^i A^j \rho\sigma^i\sigma^j\frac{\partial^2 J(A^i, A^j, k^i, k^j)}{\partial A^i \partial A^j}
\end{aligned}
$$

with boundary conditions

$$
\begin{aligned}
J(A^i, A^j, k^i, k^j) &= J(A^i, A^j, k^i + \mathbf{d}k, k^j - \mathbf{d}k) - cA^i\mathbf{d}k \\
J(A^i, A^j, k^i, k^j) &= J(A^i, A^j, k^i - \mathbf{d}k, k^j + \mathbf{d}k) - cA^j\mathbf{d}k
\end{aligned}
$$

and

$$
\begin{aligned}
&\frac{\partial J(A^i, A^j, k^i, k^j)}{\partial A^i} + \frac{\partial J(A^i, A^j, k^i, k^j)}{\partial A^j} = \\
&\qquad \frac{\partial J(A^i, A^j, k^i + \mathbf{d}k, k^j - \mathbf{d}k)}{\partial A^i} + \frac{\partial J(A^i, A^j, k^i + \mathbf{d}k, k^j - \mathbf{d}k)}{\partial A^j} - c \\
&\frac{\partial J(A^i, A^j, k^i, k^j)}{\partial A^i} + \frac{\partial J(A^i, A^j, k^i, k^j)}{\partial A^j} = \\
&\qquad \frac{\partial J(A^i, A^j, k^i - \mathbf{d}k, k^j + \mathbf{d}k)}{\partial A^i} + \frac{\partial J(A^i, A^j, k^i - \mathbf{d}k, k^j + \mathbf{d}k)}{\partial A^j} - c
\end{aligned}
$$

Using a change of variable $s = \log(A^j) - \log(A^i)$ so that $J(A^i, A^j) = A^i H(\log(A^j/A^i))$,

$$
\begin{aligned}
&\frac{\partial J}{\partial A^i} = H(s) - H'(s) \quad ; \quad \frac{\partial^2 J}{\partial (A^i)^2} = \frac{1}{A^i}\left(H''(s) - H'(s)\right) \\
&\frac{\partial J}{\partial A^j} = \frac{A^i}{A^j}H'(s) \quad ; \quad \frac{\partial^2 J}{\partial (A^j)^2} = \frac{A^i}{A^j}\left(H''(s) - H'(s)\right) \quad ; \quad \frac{\partial^2 J}{\partial A^j A^i} = \frac{1}{A^j}(H'(s) - H''(s))
\end{aligned}
$$

Substituting,

$$\overbrace{(r-\mu^i+\kappa^{(1,0)}\sigma^i)}^{\doteq\tilde{r}}H(s)=\Big[\alpha(z^i)^\varepsilon+(1-\alpha)(k^i)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}+\Big[\alpha(z^j)^\varepsilon+(1-\alpha)(k^j)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}e^s+$$
$$\underbrace{\left(\mu^j-\mu^i-\kappa^{(0,1)}\sigma^j+\kappa^{(1,0)}\sigma^i-\frac{1}{2}(\sigma^i)^2-\frac{1}{2}(\sigma^j)^2+\rho\sigma^i\sigma^j\right)}_{\doteq\tilde{u}}H'(s)+\frac{1}{2}\overbrace{((\sigma^i)^2+(\sigma^j)^2-2\rho\sigma^i\sigma^j)}^{\doteq\tilde{\sigma}^2}H''(s)$$

with boundary conditions

$$\frac{\partial H(\bar{s},k^i,k^j)}{\partial k^j}-\frac{\partial H(\bar{s},k^i,k^j)}{\partial k^i}=c\quad;\quad\frac{\partial H(\underline{s},k^i,k^j)}{\partial k^i}-\frac{\partial H(\underline{s},k^i,k^j)}{\partial k^j}=ce^{\underline{s}}\tag{A1}$$

and

$$\begin{aligned}\frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^j\partial s}-\frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^i\partial s}&=0\\ \frac{\partial^2 H(\underline{s},k^i,k^j)}{\partial k^i\partial s}-\frac{\partial^2 H(\underline{s},k^i,k^j)}{\partial k^j\partial s}&=ce^{\underline{s}}\end{aligned}\tag{A2}$$

The general solution is

$$\begin{aligned}H(s,k^i,k^j)=\frac{\Big[\alpha(z^i)^\varepsilon+(1-\alpha)(k^i)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^i}&+\frac{\Big[\alpha(z^j)^\varepsilon+(1-\alpha)(k^j)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^j}e^s\\&+\mathbb{C}_1(k^i,k^j)e^{\phi^+s}+\mathbb{C}_2(k^i,k^j)e^{\phi^-s}\end{aligned}$$

where $\delta^i\doteq r-\mu^i+\kappa^{(1,0)}\sigma^i$, $\delta^j\doteq r-\mu^j+\kappa^{(0,1)}\sigma^j$ and

$$\phi^\pm=\frac{-\tilde{u}\pm\sqrt{2\tilde{r}\tilde{\sigma}^2+\tilde{u}^2}}{\tilde{\sigma}^2}$$

To pin down $\mathbb{C}_1(k^i,k^j)$ and $\mathbb{C}_2(k^i,k^j)$, apply the boundary conditions, $\mathbb{C}_1(0,k^j)=0$ and $\mathbb{C}_2(k^i,0)=0$, or that the option value is zero in the corresponding direction when capital is exhausted. Define

$$\begin{aligned}\Upsilon_{1,1}&\doteq\frac{e^{\phi^-\bar{s}}-e^{\phi^-\underline{s}}}{e^{\phi^-\bar{s}+\phi^+\underline{s}}-e^{\phi^+\bar{s}+\phi^-\underline{s}}}\quad;\quad\Upsilon_{2,1}\doteq\frac{e^{\phi^+\underline{s}}-e^{\phi^+\bar{s}}}{e^{\phi^-\bar{s}+\phi^+\underline{s}}-e^{\phi^+\bar{s}+\phi^-\underline{s}}}\\ \Upsilon_{1,2}&\doteq\frac{e^{\phi^-\bar{s}+\underline{s}}-e^{\bar{s}+\phi^-\underline{s}}}{e^{\phi^-\bar{s}+\phi^+\underline{s}}-e^{\phi^+\bar{s}+\phi^-\underline{s}}}\quad;\quad\Upsilon_{2,2}\doteq\frac{e^{\bar{s}+\phi^+\underline{s}}-e^{\phi^+\bar{s}+\underline{s}}}{e^{\phi^-\bar{s}+\phi^+\underline{s}}-e^{\phi^+\bar{s}+\phi^-\underline{s}}}\\ \Upsilon_{1,3}&\doteq\frac{e^{\phi^-\underline{s}}-e^{\phi^-\bar{s}+\underline{s}}}{e^{\phi^-\bar{s}+\phi^+\underline{s}}-e^{\phi^+\bar{s}+\phi^-\underline{s}}}\quad;\quad\Upsilon_{2,3}\doteq\frac{e^{\phi^+\underline{s}}-e^{\phi^+\bar{s}+\underline{s}}}{e^{\phi^-\bar{s}+\phi^+\underline{s}}-e^{\phi^+\bar{s}+\phi^-\underline{s}}}\end{aligned}\tag{A3}$$

Then,

$$\mathbb{C}_1(k^i,k^j) = -\left[\frac{\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i} - \frac{[\alpha(z^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}}{\delta^i}\right]\Upsilon_{1,1}$$
$$+\left[\frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^i+k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j} - \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j}\right]\Upsilon_{1,2} - k^i c\Upsilon_{1,3}$$

$$\mathbb{C}_2(k^i,k^j) = \left[\frac{\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i+k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i} - \frac{\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i}\right]\Upsilon_{2,1}$$
$$-\left[\frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j} - \frac{[\alpha(z^j)^\varepsilon]^{\frac{\theta}{\varepsilon}}}{\delta^j}\right]\Upsilon_{2,2} - k^j c\Upsilon_{2,3} \tag{A4}$$

A.2 Proof of Corollary 1

With single direction transfer from $i \to j$, start from the general solution

$$H(s,k^i,k^j) = \frac{z^i k^i}{\delta^i} + \frac{z^j k^j}{\delta^j}e^s + \mathbb{C}_1(k^i,k^j)e^{\phi^+ s} + \mathbb{C}_2(k^i,k^j)e^{\phi^- s}$$

Since the value of the option goes to zero when $s \to -\infty$, $\mathbb{C}_2(k^i,k^j) = 0$. Moreover,

$$\frac{\partial H(\bar{s},k^i,k^j)}{\partial k^j} - \frac{\partial H(\bar{s},k^i,k^j)}{\partial k^i} = c \tag{A5}$$
$$\frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^j \partial s} - \frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^i \partial s} = 0 \tag{A6}$$

Applying (A5) and (A6) leads to (11).

A.3 Proof of Proposition 2

The firm's problem is now

$$
\begin{aligned}
\max_{\{\Delta k_t^{i\to j},\Delta k_t^{j\to i},\tau\}} \quad & \mathbb{E}_0^{\mathbb{Q}}\Bigg[\int_0^{\tau} e^{-rt}\Bigg(\Big[\alpha(z^i)^{\varepsilon}+(1-\alpha)(k_t^i)^{\varepsilon}\Big]^{\frac{\theta}{\varepsilon}}A_t^i\Bigg)\mathbf{d}t\Bigg] \\
& +\mathbb{E}_0^{\mathbb{Q}}\Bigg[\int_{\tau}^{\infty} e^{-rt}\Bigg(\Big[\alpha(z^i+\mathfrak{z}^i)^{\varepsilon}+(1-\alpha)(k_t^i+\mathscr{k}^i)^{\varepsilon}\Big]^{\frac{\theta}{\varepsilon}}A_t^i\Bigg)\mathbf{d}t\Bigg] \\
& +\mathbb{E}_0^{\mathbb{Q}}\Bigg[\int_0^{\infty} e^{-rt}\Big[\alpha(z^j)^{\varepsilon}+(1-\alpha)(k_t^j)^{\varepsilon}\Big]^{\frac{\theta}{\varepsilon}}A_t^j\mathbf{d}t\Bigg] \\
& -e^{-r\tau}\varpi^i\mathbb{E}_0^{\mathbb{Q}}\Big[A_\tau^i\Big]-\mathbb{E}_0^{\mathbb{Q}}\left[\int_0^{\infty} e^{-rt}\left(cA_t^i\Delta k_t^{i\to j}+cA_t^j\Delta k_t^{j\to i}\right)\mathbf{d}t\right]
\end{aligned}
$$

such that

$$
\begin{aligned}
& k_t^i+k_t^j=K \\
& k_t^i\geq 0, k_t^j\geq 0 \\
& k_t^i=k_{t^-}^i-\Delta k_t^{i\to j}+\Delta k_t^{j\to i} \\
& k_t^j=k_{t^-}^j-\Delta k_t^{j\to i}+\Delta k_t^{i\to j}
\end{aligned}
$$

Applying the change of variable $s=\log A^j-\log A^i$, the HJB equation solves the boundary conditions for investment

$$
H(\widehat{s},k^i,k^j,z^i)=H(\widehat{s},k^i+\mathscr{k}^i,k^j,z^i+\mathfrak{z}^i)-\varpi^i \tag{A7}
$$

$$
\frac{\partial H(\widehat{s},k^i,k^j,z^i)}{\partial s}=\frac{\partial H(\widehat{s},k^i+\mathscr{k}^i,k^j,z^i+\mathfrak{z}^i)}{\partial s} \tag{A8}
$$

and boundary conditions for reallocation before investment

$$
\frac{\partial H(\bar{s},k^i,k^j,z^i)}{\partial k^j}-\frac{\partial H(\bar{s},k^i,k^j,z^i)}{\partial k^i}=c \tag{A9}
$$

$$
\frac{\partial H(\underline{s},k^i,k^j,z^i)}{\partial k^i}-\frac{\partial H(\underline{s},k^i,k^j,z^i)}{\partial k^j}=ce^{\underline{s}} \tag{A10}
$$

$$
\frac{\partial^2 H(\bar{s},k^i,k^j,z^i)}{\partial k^j\partial s}-\frac{\partial^2 H(\bar{s},k^i,k^j,z^i)}{\partial k^i\partial s}=0 \tag{A11}
$$

$$
\frac{\partial^2 H(\underline{s},k^i,k^j,z^i)}{\partial k^i\partial s}-\frac{\partial^2 H(\underline{s},k^i,k^j,z^i)}{\partial k^j\partial s}=ce^{\underline{s}} \tag{A12}
$$

and boundary conditions for reallocation after investment

$$
\begin{aligned}
&\frac{\partial H(\bar{\bar{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^j} - \frac{\partial H(\bar{\bar{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^i} = c \\
&\frac{\partial H(\underline{\underline{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^i} - \frac{\partial H(\underline{\underline{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^j} = ce^{\underline{\underline{s}}} \\
&\frac{\partial^2 H(\bar{\bar{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^j \partial s} - \frac{\partial^2 H(\bar{\bar{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^i \partial s} = 0 \\
&\frac{\partial^2 H(\underline{\underline{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^i \partial s} - \frac{\partial^2 H(\underline{\underline{s}}, k^i + \not{k}^i, k^j, z^i + \mathfrak{z}^i)}{\partial k^j \partial s} = ce^{\underline{\underline{s}}} \qquad \text{(A13)}
\end{aligned}
$$

The reallocation boundaries $\bar{\bar{s}}, \underline{\underline{s}}$ emphasize the fact that reallocation barriers change after the firm invests.

Using conditions A7, A9 and A10, firm value is

$$
\begin{aligned}
H(s, k^i, k^j, z) = &\frac{\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i} + \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j} e^s \\
&+ \mathbb{C}_1(k^i, k^j) e^{\phi^+ s} + \mathbb{D}_2(k^i + \not{k}^i, k^j) e^{\phi^- s} \\
&+ \left[\mathbb{D}_1(k^i + \not{k}^i, k^j) - \mathbb{C}_1(k^i, k^j)\right] e^{\phi^- s + (\phi^+ - \phi^-)\widehat{s}} + \text{Surplus}^i e^{\phi^-(s - \widehat{s})} \qquad \text{(A14)}
\end{aligned}
$$

where

$$
\text{Surplus}^i = [\alpha(z^i + \mathfrak{z}^i)^\varepsilon + (1-\alpha)(k^i + \not{k}^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}/\delta^i - [\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}/\delta^i - \varpi^i
$$

and

$$
\begin{aligned}
\mathbb{C}_1(k^i, k^j) = &\Bigg[y^j e^{\widehat{s}} - y^i e^{\phi^-(\bar{s} - \widehat{s})} - \left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}/\delta^i + [\alpha(z^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}/\delta^i - ck^i \\
&- \mathbb{D}_1(k^i + \not{k}^i, k^j) e^{\phi^- \bar{s} + (\phi^+ - \phi^-)\widehat{s}} - \mathbb{D}_2(k^i + \not{k}^i, k^j) e^{\phi^- \bar{s}} \Bigg] \Big/ \left(e^{\phi^+ \bar{s}} - e^{\phi^- \bar{s} + (\phi^+ - \phi^-)\widehat{s}} \right) \qquad \text{(A15)}
\end{aligned}
$$

using

$$
\begin{aligned}
y^i \doteq &\left[\alpha(z^i + \mathfrak{z}^i)^\varepsilon + (1-\alpha)(k^i + \not{k}^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}/\delta^i - \left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}/\delta^i \\
&- \left[\alpha(z^i + \mathfrak{z}^i)^\varepsilon + (1-\alpha)(\not{k}^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}/\delta^i + [\alpha(z^i)^\varepsilon]^{\frac{\theta}{\varepsilon}}/\delta^i \\
y^j \doteq &\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^i + k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}/\delta^j - \left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}/\delta^j
\end{aligned}
$$

Moreover, $\mathbb{D}_1(k^i + \not{k}^i, k^j), \mathbb{D}_2(k^i + \not{k}^i, k^j)$ are given by Equation A4 but changing the function arguments and using new constants,

$$\begin{aligned}
\Psi_{1,1} &\doteq \frac{e^{\phi^- \bar{\bar{s}}} - e^{\phi^- \underline{\underline{s}}}}{e^{\phi^- \bar{\bar{s}} + \phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \phi^- \underline{\underline{s}}}} \quad ; \quad \Psi_{2,1} \doteq \frac{e^{\phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}}}}{e^{\phi^- \bar{\bar{s}} + \phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \phi^- \underline{\underline{s}}}} \\
\Psi_{1,2} &\doteq \frac{e^{\phi^- \bar{\bar{s}} + \underline{\underline{s}}} - e^{\bar{\bar{s}} + \phi^- \underline{\underline{s}}}}{e^{\phi^- \bar{\bar{s}} + \phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \phi^- \underline{\underline{s}}}} \quad ; \quad \Psi_{2,2} \doteq \frac{e^{\bar{\bar{s}} + \phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \underline{\underline{s}}}}{e^{\phi^- \bar{\bar{s}} + \phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \phi^- \underline{\underline{s}}}} \\
\Psi_{1,3} &\doteq \frac{e^{\phi^- \underline{\underline{s}}} - e^{\phi^- \bar{\bar{s}} + \underline{\underline{s}}}}{e^{\phi^- \bar{\bar{s}} + \phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \phi^- \underline{\underline{s}}}} \quad ; \quad \Psi_{2,3} \doteq \frac{e^{\phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \underline{\underline{s}}}}{e^{\phi^- \bar{\bar{s}} + \phi^+ \underline{\underline{s}}} - e^{\phi^+ \bar{\bar{s}} + \phi^- \underline{\underline{s}}}} \qquad \text{(A16)}
\end{aligned}$$

where the post-investment boundaries are pinned down via the conditions in A13.

Applying A8 along with A11 and A12 completes the characterization of the investment and pre-investment reallocation thresholds. The semi-explicit solution for $\widehat{s}$ in Equation 17 is a direction application of A8 following A7.

A.4 Proof of Proposition 3

With CES production technology, the modified HJB equation in the no-action region is given by

$$
\begin{aligned}
\tilde{r}H(s,k^i,k^j,z^i,z^j) = {} & \Big[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}} + \Big[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}} e^s \\
& + \tilde{\mu}H'(s,k^i,k^j,z^i,z^j) + .5\tilde{\sigma}^2 H''(s,k^i,k^j,z^i,z^j) \\
& + \lambda^i\Big[H(s,k^i+\tilde{\mathscr{k}},k^j,z^i+\tilde{\mathscr{z}}^i,z^j) - H(s,k^i,k^j,z^i,z^j) - \varpi^i\Big] \\
& + \lambda^j\Big[H(s,k^i,k^j+\tilde{\mathscr{k}},z^i,z^j+\tilde{\mathscr{z}}^j) - H(s,k^i,k^j,z^i,z^j) - \varpi^j e^s\Big]
\end{aligned}
\tag{A17}
$$

where H' and H'' are derivatives with respect to s. There are five value functions. 1, the current regime is a i project arrival. 2, the current regime is a j project arrival. 3, the project was immediately adopted and the firm is currently in waiting. 4, in expectation, the firm no longer expects to adopt a project in i and 5, in expectation, the firm no longer expects to adopt a project in j. To determine optimal investment boundaries for i and j projects, only the first three are relevant.

In the 3rd regime, the general solution is

$$
\begin{aligned}
H(s,k^i,k^j,z^i,z^j) = {} & \frac{\Big[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^i+\lambda^i+\lambda^j} + \frac{\Big[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^i+\lambda^i+\lambda^j} e^s \\
& + \overbrace{\mathbb{C}_1(k^i,k^j)e^{\phi_1^+ s}}^{\substack{\text{option to transfer} \\ \text{from } i\to j}} + \overbrace{\mathbb{C}_2(k^i,k^j)e^{\phi_1^- s}}^{\substack{\text{option to transfer} \\ \text{from } j\to i}} \\
& + \frac{\lambda^i}{\delta^i+\lambda^i+\lambda^j} \overbrace{\frac{\Big[\alpha(z^i+\tilde{\mathscr{z}}^i)^\varepsilon + (1-\alpha)(k^i+\tilde{\mathscr{k}}^i)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^i}}^{\substack{\text{change to production technology} \\ \text{after project } i \text{ is adopted}}} + \frac{\lambda^i}{\delta^j+\lambda^i+\lambda^j} \frac{\Big[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^j} e^s \\
& + \frac{\lambda^i}{\lambda^i+\lambda^j} \overbrace{\Big[\mathbb{C}_1^i(k^i+\tilde{\mathscr{k}}^i,k^j)e^{\phi_0^+ s} + \mathbb{C}_2^i(k^i+\tilde{\mathscr{k}}^i,k^j)e^{\phi_0^- s}\Big]}^{\substack{\text{new option values if project } i \\ \text{arrives and is adopted}}} \\
& + \frac{\lambda^j}{\delta^i+\lambda^i+\lambda^j} \frac{\Big[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^i} + \frac{\lambda^j}{\delta^j+\lambda^i+\lambda^j} \frac{\Big[\alpha(z^j+\tilde{\mathscr{z}}^j)^\varepsilon + (1-\alpha)(k^j+\tilde{\mathscr{k}}^j)^\varepsilon\Big]^{\frac{\theta}{\varepsilon}}}{\delta^j} e^s \\
& + \frac{\lambda^j}{\lambda^i+\lambda^j} \Big[\mathbb{C}_1^j(k^i,k^j+\tilde{\mathscr{k}}^j)e^{\phi_0^+ s} + \mathbb{C}_2^j(k^i,k^j+\tilde{\mathscr{k}}^j)e^{\phi_0^- s}\Big] \\
& \underbrace{-\frac{\lambda^i}{\delta^i+\lambda^i+\lambda^j}\varpi^i - \frac{\lambda^j}{\delta^j+\lambda^i+\lambda^j}\varpi^j e^s}_{\text{project costs}}
\end{aligned}
\tag{A18}
$$

where

$$\phi_0^{\pm} = \frac{-\tilde{\mu} \pm \sqrt{2\tilde{r}\tilde{\sigma}^2 + \tilde{\mu}^2}}{\tilde{\sigma}^2}$$
$$\phi_1^{\pm} = \frac{-\tilde{\mu} \pm \sqrt{2(\tilde{r} + \lambda^i + \lambda^j)\tilde{\sigma}^2 + \tilde{\mu}^2}}{\tilde{\sigma}^2}$$

The boundary conditions for (A18) are: first, continuity,

$$\begin{aligned}
&\frac{\partial H(\underline{s}, k^i, k^j, z^i, z^j)}{\partial k^i} - \frac{\partial H(\underline{s}, k^i, k^j, z^i, z^j)}{\partial k^j} = ce^{\underline{s}} \\
&\frac{\partial H(\bar{s}, k^i, k^j, z^i, z^j)}{\partial k^j} - \frac{\partial H(\bar{s}, k^i, k^j, z^i, z^j)}{\partial k^i} = c \\
&\frac{\partial H(\underline{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^i} - \frac{\partial H(\underline{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^j} = ce^{\underline{s}^i} \\
&\frac{\partial H(\bar{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^j} - \frac{\partial H(\bar{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^i} = c \\
&\frac{\partial H(\underline{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^i} - \frac{\partial H(\underline{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^j} = ce^{\underline{s}^j} \\
&\frac{\partial H(\bar{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^j} - \frac{\partial H(\bar{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^i} = c
\end{aligned} \tag{A19}$$

then, optimality,

$$\begin{aligned}
&\frac{\partial^2 H(\underline{s}, k^i, k^j, z^i, z^j)}{\partial k^i \partial s} - \frac{\partial^2 H(\underline{s}, k^i, k^j, z^i, z^j)}{\partial k^j \partial s} = ce^{\underline{s}} \\
&\frac{\partial^2 H(\bar{s}, k^i, k^j, z^i, z^j)}{\partial k^j \partial s} - \frac{\partial^2 H(\bar{s}, k^i, k^j, z^i, z^j)}{\partial k^i \partial s} = 0 \\
&\frac{\partial^2 H(\underline{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^i \partial s} - \frac{\partial^2 H(\underline{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^j \partial s} = ce^{\underline{s}^i} \\
&\frac{\partial^2 H(\bar{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^j \partial s} - \frac{\partial^2 H(\bar{s}^i, k^i + \tilde{\mathscr{k}}, k^j, z^i + \tilde{\mathscr{z}}^i, z^j)}{\partial k^i \partial s} = 0 \\
&\frac{\partial^2 H(\underline{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^i \partial s} - \frac{\partial^2 H(\underline{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^j \partial s} = ce^{\underline{s}^j} \\
&\frac{\partial^2 H(\bar{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^j \partial s} - \frac{\partial^2 H(\bar{s}^j, k^i, k^j + \tilde{\mathscr{k}}, z^i, z^j + \tilde{\mathscr{z}}^j)}{\partial k^i \partial s} = 0
\end{aligned} \tag{A20}$$

which explicitly pin down the six integration constants $\mathbb{C}_1(k^i, k^j)$, $\mathbb{C}_2(k^i, k^j)$ and $\mathbb{C}_1^i(k^i + \tilde{\mathscr{k}}^i, k^j)$, $\mathbb{C}_2^i(k^i + \tilde{\mathscr{k}}^i, k^j)$, $\mathbb{C}_1^j(k^i, k^j + \tilde{\mathscr{k}}^j)$, $\mathbb{C}_2^j(k^i, k^j + \tilde{\mathscr{k}}^j)$.

For the base form of $\mathbb{C}_1(k^i,k^j),\ \mathbb{C}_2(k^i,k^j)$, define

$$\begin{aligned}
\Upsilon_{1,1} &\doteq \frac{e^{\phi_1^-\bar{s}}-e^{\phi_1^-\underline{s}}}{e^{\phi_1^-\bar{s}+\phi_1^+\underline{s}}-e^{\phi_1^+\bar{s}+\phi_1^-\underline{s}}} \quad ; \quad \Upsilon_{2,1} \doteq \frac{e^{\phi_1^+\bar{s}}-e^{\phi_1^+\underline{s}}}{e^{\phi_1^-\bar{s}+\phi_1^+\underline{s}}-e^{\phi_1^+\bar{s}+\phi_1^-\underline{s}}} \\
\Upsilon_{1,2} &\doteq \frac{e^{\bar{s}+\phi_1^-\underline{s}}-e^{\phi_1^-\bar{s}+\underline{s}}}{e^{\phi_1^-\bar{s}+\phi_1^+\underline{s}}-e^{\phi_1^+\bar{s}+\phi_1^-\underline{s}}} \quad ; \quad \Upsilon_{2,2} \doteq \frac{e^{\bar{s}+\phi_1^+\underline{s}}-e^{\phi_1^+\bar{s}+\underline{s}}}{e^{\phi_1^-\bar{s}+\phi_1^+\underline{s}}-e^{\phi_1^+\bar{s}+\phi_1^-\underline{s}}} \\
\Upsilon_{1,3} &\doteq \frac{e^{\phi_1^-\bar{s}+\underline{s}}+e^{\phi_1^-\underline{s}}}{e^{\phi_1^-\bar{s}+\phi_1^+\underline{s}}-e^{\phi_1^+\bar{s}+\phi_1^-\underline{s}}} \quad ; \quad \Upsilon_{2,3} \doteq \frac{e^{\phi_1^+\bar{s}+\underline{s}}+e^{\phi_1^+\underline{s}}}{e^{\phi_1^-\bar{s}+\phi_1^+\underline{s}}-e^{\phi_1^+\bar{s}+\phi_1^-\underline{s}}}
\end{aligned} \tag{A21}$$

Moreover, let

$$\begin{aligned}
v_i &= \frac{[\alpha(z^i)^\varepsilon+(1-\alpha)(k^i)^\varepsilon]^{\theta/\varepsilon}}{\delta^i+\lambda^i+\lambda^j} \\
v_{ii} &= \frac{\lambda^i}{\delta^i+\lambda^i+\lambda^j}\frac{[\alpha(z^i+\mathfrak{z}^i)^\varepsilon+(1-\alpha)(k^i+\not{k}^i)^\varepsilon]^{\theta/\varepsilon}}{\delta^i} \\
v_{ij} &= \frac{\lambda^j}{\delta^i+\lambda^i+\lambda^j}\frac{[\alpha(z^i)^\varepsilon+(1-\alpha)(k^i)^\varepsilon]^{\theta/\varepsilon}}{\delta^i} \\
v_j &= \frac{[\alpha(z^j)^\varepsilon+(1-\alpha)(k^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j+\lambda^i+\lambda^j} \\
v_{jj} &= \frac{\lambda^j}{\delta^j+\lambda^i+\lambda^j}\frac{[\alpha(z^j+\mathfrak{z}^j)^\varepsilon+(1-\alpha)(k^j+\not{k}^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j} \\
v_{jj} &= \frac{\lambda^i}{\delta^j+\lambda^i+\lambda^j}\frac{[\alpha(z^j)^\varepsilon+(1-\alpha)(k^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j} \\
x_i &= \frac{[\alpha(z^i)^\varepsilon+(1-\alpha)(k^i+k^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^i+\lambda^i+\lambda^j}+\frac{\lambda^i}{\delta^i+\lambda^i+\lambda^j}\frac{[\alpha(z^i+\mathfrak{z}^i)^\varepsilon+(1-\alpha)(k^i+k^j+\not{k}^i)^\varepsilon]^{\theta/\varepsilon}}{\delta^i} \\
&\qquad +\frac{\lambda^j}{\delta^i+\lambda^i+\lambda^j}\frac{[\alpha(z^i)^\varepsilon+(1-\alpha)(k^i+k^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^i} \\
x_j &= \frac{[\alpha(z^j)^\varepsilon+(1-\alpha)(k^i+k^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j+\lambda^i+\lambda^j}+\frac{\lambda^j}{\delta^j+\lambda^i+\lambda^j}\frac{[\alpha(z^j+\mathfrak{z}^j)^\varepsilon+(1-\alpha)(k^i+k^j+\not{k}^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j} \\
&\qquad +\frac{\lambda^i}{\delta^j+\lambda^i+\lambda^j}\frac{[\alpha(z^j)^\varepsilon+(1-\alpha)(k^i+k^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j} \\
adjz^i &= \frac{[\alpha(z^i)^\varepsilon]^{\theta/\varepsilon}}{\delta^i+\lambda^i+\lambda^j}+\frac{\lambda^i}{\delta^i+\lambda^i+\lambda^j}\frac{[\alpha(z^i+\mathfrak{z}^i)^\varepsilon+(1-\alpha)(\not{k}^i)^\varepsilon]^{\theta/\varepsilon}}{\delta^i}+\frac{\lambda^j}{\delta^i+\lambda^i+\lambda^j}\frac{[\alpha(z^i)^\varepsilon]^{\theta/\varepsilon}}{\delta^i} \\
adjz^j &= \frac{[\alpha(z^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j+\lambda^i+\lambda^j}+\frac{\lambda^i}{\delta^j+\lambda^i+\lambda^j}\frac{[\alpha(z^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j}+\frac{\lambda^j}{\delta^j+\lambda^i+\lambda^j}\frac{[\alpha(z^j+\mathfrak{z}^j)^\varepsilon+(1-\alpha)(\not{k}^j)^\varepsilon]^{\theta/\varepsilon}}{\delta^j}
\end{aligned}$$

Then,

$$\begin{aligned}
\mathbb{C}_1(k^i,k^j) &= -(vi+vii+vij-adjz^i)\Upsilon_{1,1}+(vj+vjj+vji-xj)\Upsilon_{1,2}+ck^i\Upsilon_{1,3} \\
\mathbb{C}_2(k^i,k^j) &= (vi+vii+vij-xi)\Upsilon_{2,1}-(vj+vjj+vji-adjz^j)\Upsilon_{2,2}+ck^j\Upsilon_{2,3}
\end{aligned} \tag{A22}$$

The two equations in A22 can then be adapted to accommodate the other regimes via superposition.

For the remaining four integration constants, they take identical functional forms as in Equation A4, but replacing the function arguments with the appropriate distribution of general assets. Verification easily follows.

To find project adoption thresholds, suppose a project i arrives. The composition is drawn and $\mathfrak{z}^i$ and $\mathfrak{k}^i$ are known, as opposed to the random variables $\tilde{\mathfrak{z}}^i$ and $\tilde{\mathfrak{k}}^i$. If the project is adopted, the post-adoption value function is then

$$
\begin{aligned}
H(s, k^i + \mathfrak{k}, k^j, z^i + \mathfrak{z}^i, z^j) = {} & \frac{\left[\alpha(z^i + \mathfrak{z}^i)^\varepsilon + (1-\alpha)(k^i + \mathfrak{k}^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i + \lambda^i + \lambda^j} + \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i + \lambda^i + \lambda^j} e^s \\
& + \mathbb{D}_1(k^i + \mathfrak{k}^i, k^j) e^{\phi_1^+ s} + \mathbb{D}_2(k^i + \mathfrak{k}^i, k^j) e^{\phi_1^- s} \\
& + \frac{\lambda^i}{\delta^i + \lambda^i + \lambda^j} \frac{\left[\alpha(z^i + \mathfrak{z}^i + \tilde{\mathfrak{z}}^i)^\varepsilon + (1-\alpha)(k^i + \mathfrak{k}^i + \tilde{\mathfrak{k}}^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i} \\
& + \frac{\lambda^i}{\delta^j + \lambda^i + \lambda^j} \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j} e^s \\
& + \frac{\lambda^i}{\lambda^i + \lambda^j} \left[\mathbb{D}_1^i(k^i + \mathfrak{k}^i + \tilde{\mathfrak{k}}^i, k^j) e^{\phi_0^+ s} + \mathbb{D}_2^i(k^i + \mathfrak{k}^i + \tilde{\mathfrak{k}}^i, k^j) e^{\phi_0^- s}\right] \\
& + \frac{\lambda^j}{\delta^i + \lambda^i + \lambda^j} \frac{\left[\alpha(z^i + \mathfrak{z}^i)^\varepsilon + (1-\alpha)(k^i + \mathfrak{k}^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i} \\
& + \frac{\lambda^j}{\delta^j + \lambda^i + \lambda^j} \frac{\left[\alpha(z^j + \tilde{\mathfrak{z}}^j)^\varepsilon + (1-\alpha)(k^j + \tilde{\mathfrak{k}}^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j} e^s \\
& + \frac{\lambda^j}{\lambda^i + \lambda^j} \left[\mathbb{D}_1^j(k^i + \mathfrak{k}^i, k^j + \tilde{\mathfrak{k}}^j) e^{\phi_0^+ s} + \mathbb{D}_2^j(k^i + \mathfrak{k}^i, k^j + \tilde{\mathfrak{k}}^j) e^{\phi_0^- s}\right] \\
& - \frac{\lambda^i}{\delta^i + \lambda^i + \lambda^j} \varpi^i - \frac{\lambda^j}{\delta^j + \lambda^i + \lambda^j} \varpi^j e^s
\end{aligned} \tag{A23}
$$

and boundary conditions written similar to the ones above. Moreover,

$$H(\hat{s}^*, k^i, k^j, z^i, z^j) = H(\hat{s}^*, k^i + \mathfrak{k}^i, k^j, z^i + \mathfrak{z}^i, z^j) - \varpi^i \tag{A24}$$

$$\frac{\partial H(\hat{s}^*, k^i, k^j, z^i, z^j)}{\partial s} = \frac{\partial H(\hat{s}^*, k^i + \mathfrak{k}^i, k^j, z^i + \mathfrak{z}^i, z^j)}{\partial s} \tag{A25}$$

Applying boundary conditions A24 and A25,

$$
\begin{aligned}
&\phi_1^- \times \text{Surplus}^i \\
&\quad = (\phi_0^+ - \phi_1^-)\Bigg[\frac{\lambda^i}{\lambda^i+\lambda^j}\Big(\mathbb{D}_1^i(k^i + \hbar^i + \tilde{\hbar}^i, k^j) - \mathbb{C}_1^i(k^i + \tilde{\hbar}^i, k^j)\Big) \\
&\qquad\qquad + \frac{\lambda^j}{\lambda^i+\lambda^j}\Big(\mathbb{D}_1^j(k^i + \hbar^i, k^j + \tilde{\hbar}^j) - \mathbb{C}_1^j(k^i, k^j + \tilde{\hbar}^j)\Big)\Bigg] e^{\phi_0^+ \hat{s}^*} \\
&\quad + (\phi_0^- - \phi_1^-)\Bigg[\frac{\lambda^i}{\lambda^i+\lambda^j}\Big(\mathbb{D}_2^i(k^i + \hbar^i + \tilde{\hbar}^i, k^j) - \mathbb{C}_2^i(k^i + \tilde{\hbar}^i, k^j)\Big) \\
&\qquad\qquad + \frac{\lambda^j}{\lambda^i+\lambda^j}\Big(\mathbb{D}_2^j(k^i + \hbar^i, k^j + \tilde{\hbar}^j) - \mathbb{C}_2^j(k^i, k^j + \tilde{\hbar}^j)\Big)\Bigg] e^{\phi_0^- \hat{s}^*} \\
&\quad + (\phi_1^+ - \phi_1^-)\left[\mathbb{D}_1(k^i + \hbar^i, k^j) - \mathbb{C}_1(k^i, k^j)\right] e^{\phi_1^+ \hat{s}^*} \qquad \text{(A26)}
\end{aligned}
$$

which can be solved numerically for $\hat{s}^*, \bar{s}, \underline{s}$.

Furthermore, for a project in j, we can do the same by writing $\mathbb{C}_4$ and apply the boundary conditions

$$H(\check{s}^*, k^i, k^j, z^i, z^j) = H(\check{s}^*, k^i, k^j + \hbar^j, z^i, z^j + \mathfrak{z}^j) - \varpi^i e^{\check{s}^*} \qquad \text{(A27)}$$

$$\frac{\partial H(\check{s}^*, k^i, k^j, z^i, z^j)}{\partial s} = \frac{\partial H(\check{s}^*, k^i + \hbar^i, k^j, z^i + \mathfrak{z}^i, z^j)}{\partial s} - \varpi^j e^{\check{s}^*} \qquad \text{(A28)}$$

to find the optimal project adoption threshold $\check{s}^*$ for j,

$$
\begin{aligned}
&0 = (1 - \phi_1^+) \times \text{Surplus}^j \\
&\quad + (\phi_0^+ - \phi_1^+)\Bigg[\frac{\lambda^i}{\lambda^i+\lambda^j}\Big(\mathbb{D}_1^i(k^i + \tilde{\hbar}^i, k^j + \hbar^j) - \mathbb{C}_1^i(k^i + \tilde{\hbar}^i, k^j)\Big) \\
&\qquad\qquad + \frac{\lambda^j}{\lambda^i+\lambda^j}\Big(\mathbb{D}_1^j(k^i, k^j + \hbar^j + \tilde{\hbar}^j) - \mathbb{C}_1^j(k^i, k^j + \tilde{\hbar}^j)\Big)\Bigg] e^{(\phi_0^+ - 1)\check{s}^*} \\
&\quad + (\phi_0^- - \phi_1^+)\Bigg[\frac{\lambda^i}{\lambda^i+\lambda^j}\Big(\mathbb{D}_2^i(k^i + \tilde{\hbar}^i, k^j + \hbar^j) - \mathbb{C}_2^i(k^i + \tilde{\hbar}^i, k^j)\Big) \\
&\qquad\qquad + \frac{\lambda^j}{\lambda^i+\lambda^j}\Big(\mathbb{D}_2^j(k^i, k^j + \hbar^j + \tilde{\hbar}^j) - \mathbb{C}_2^j(k^i, k^j + \tilde{\hbar}^j)\Big)\Bigg] e^{(\phi_0^- - 1)\check{s}^*} \\
&\quad + (\phi_1^- - \phi_1^+)\left[\mathbb{D}_2(k^i, k^j + \hbar^j) - \mathbb{C}_2(k^i, k^j)\right] e^{(\phi_1^- - 1)\check{s}^*} \qquad \text{(A29)}
\end{aligned}
$$

A.5 *Proof of Proposition 4*

Since the density for the distribution of passage times for Brownian motion with drift is well known, one can take the expected value of the density function directly to find the expected passage time. However, I provide a much simpler proof using the optional sampling theorem. Let X_t be the exponential martingale

$$X_t = \exp\left(\eta(s_t - s_0) - \eta\mu^s t - \frac{1}{2}(\eta\sigma^s)^2 t\right)$$

where s_0 is the initial position of s_t and assume $\eta > 0$. Moreover,

$$\begin{aligned}\mu^s &= \mu^j - \mu^i + \frac{1}{2}\left((\sigma^i)^2 - (\sigma^j)^2\right)\\ \sigma^s &= \sqrt{(\rho\sigma^j - \sigma^i)^2 + (\sigma^j)^2(1-\rho^2)}\end{aligned}$$

and B_t^* is a new Brownian motion. Define the stopped process, $(X_{t\wedge\tau})_{t\geq 0}$ where $\tau \doteq \inf\{t \geq 0 : s_t = \widehat{s}\}$. The process $(X_{t\wedge\tau})_{t\geq 0}$ is a martingale, and because it is bounded above by $\exp(\eta(\widehat{s} - s_0))$, we have by dominated convergence, $\lim_{t\to\infty}\mathbb{E}[X_{t\wedge\tau}] = \mathbb{E}[\lim_{t\to\infty} X_{t\wedge\tau}] = \mathbb{E}[X_\tau]$ so $(X_{t\wedge\tau})_{t\geq 0}$ is uniformly integrable. Applying Doob's optional sampling theorem,

$$1 = \mathbb{E}[X_0] = \mathbb{E}[X_\tau] = \mathbb{E}\left[\exp\left(\eta(\widehat{s} - s_0) - \eta\mu^s\tau - \frac{1}{2}(\eta\sigma^s)^2\tau\right)\right]$$

Rearranging,

$$\exp(\eta(s_0 - \widehat{s})) = \mathbb{E}\left[\exp\left(-\left(\eta\mu^s + \frac{1}{2}(\eta\sigma^s)^2\right)\tau\right)\right]$$

Let $\lambda = \eta\mu^s + \frac{1}{2}(\eta\sigma^s)^2$. Then, $\eta = -(\mu^s - \sqrt{2\lambda(\sigma^s)^2 + (\mu^s)^2}/(\sigma^s)^2)$ and

$$\mathbb{E}\left[e^{-\lambda\tau}\right] = e^{(\widehat{s}-s_0)(\mu^s - \sqrt{2\lambda(\sigma^s)^2 + (\mu^s)^2}/(\sigma^s)^2)} \tag{A30}$$

The above equation is the Laplace transform of τ. To find the expected first passage time, we can evaluate minus of the derivative with respect to λ at $\lambda = 0$,

$$\mathbb{E}[\tau] = -\frac{d\mathcal{L}}{d\lambda}\Big|_{\lambda=0} = \frac{\widehat{s} - s_0}{\mu^s} \quad ; \quad \mu^s \neq 0 \tag{A31}$$

which is valid for $\mu^i > \mu^j - .5(\sigma^j)^2 + .5(\sigma^i)^2$ when $\sigma^i > \sigma^j$.

If the firm is initially outside of the investment region ($s_0 > \widehat{s}$) and i is riskier than j, $\sigma^i > \sigma^j$, this lemma requires the direction of the drift to coincide with the direction of travel for the expected value of τ to be finite (μ^s must be < 0).[38] In other words, if a business pairs

[38] Karatzas and Shreve 1988 provide the density for τ to reach level $b \neq 0$,

$$f_\tau(t) = \frac{|b|}{\sqrt{2\pi t^3}}\exp\left(-\frac{(b-\mu t)^2}{2t}\right)$$

together two businesses that differ in their level of risk, then the riskier business $\sigma^i > \sigma^j$ must compensate with a higher growth rate to hit the investment threshold in expectation.[39]

To find the investment boundary for the nth investment project, apply a zero profit condition which leads to

$$\widehat{s}_n = \frac{1}{\phi^+} \log\left(\frac{\text{Surplus}_n^i}{\mathbb{C}_{1,n-1} - \mathbb{C}_{1,n}}\right) \tag{A32}$$

where $\mathbb{C}_{1,n-1}$ and $\mathbb{C}_{1,n}$ are the options value of reallocation before and after investment in the nth project (similar to $\mathbb{C}_1$ and $\mathbb{D}_1$ in 2.2), and Surplus_n^i is the benefit in production net of costs,

$$\begin{aligned}\text{Surplus}_n^i = &\left[\alpha((1+p_z)^n z^i)^\varepsilon + (1-\alpha)((1+p_k)^n k^i)^\varepsilon\right]^{\theta/\varepsilon} / \delta^i \\ &- \left[\alpha((1+p_z)^{n-1} z^i)^\varepsilon + (1-\alpha)((1+p_k)^{n-1} k^i)^\varepsilon\right]^{\theta/\varepsilon} / \delta^i \\ &- \left[\alpha(p_z(1+p_z)^{n-1} z^i)^\varepsilon + (1-\alpha)(p_k(1+p_k)^{n-1} k^i)^\varepsilon\right]^{\theta/\varepsilon} / \delta^i\end{aligned}$$

where p_z, p_k denotes the fixed proportion of specific and general assets in this project as a fraction of current capital stock and $\mathbb{C}_{1,n}$ given by,

$$\begin{aligned}\mathbb{C}_{1,n}(k^i, k^j) = &\left[\frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)((1+p_k)^n k^i + k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j} - \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^j}\right] e^{(1-\phi^+)\bar{s}_n} \\ &- \left[\frac{\left[\alpha((1+p_z)^n z^i)^\varepsilon + (1-\alpha)((1+p_k)^n k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i} - \frac{\left[\alpha((1+p_z)^n z^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^i} + c(1+p_k)^n k^i\right] e^{-\phi^+ \bar{s}_n}\end{aligned}$$

where

$$\bar{s}_n = \log\left(\frac{\phi^+}{\phi^+ - 1} \frac{\theta(1-\alpha)((1+p_k)^n k^i)^{\varepsilon-1}\left[\alpha((1+p_z)^n z^i)^\varepsilon + (1-\alpha)((1+p_k)^n k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}-1} / \delta^i + c}{\theta(1-\alpha)(k^j)^{\varepsilon-1}\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}-1} / \delta^j}\right) \tag{A33}$$

$\mathbb{C}_{1,n-1}$ is semi-explicit, and follows the steps in the derivation of Equation A15 using boundary conditions A5 and A6.

and make the identical point that $\mu \neq 0$ reaches b with probability 1 iff μ and b have the same sign; otherwise, this density is "defunct".

[39] Alternatively if $s_0 \leq \widehat{s}$ and $\mu^s > 0$, the stopping time τ would be the expected time to escape the investment region.

B Appendix: Data Construction

B.1 Data Definitions & Additional Comments

Data for financial variables runs from 1976-2022, with estimation of volatility starting in 1971. I use the Compustat variable "sich" for the firm's primary industry at fiscal year t. This variable is often missing. I replace missing values using this order of priority: 1. Most recent non-missing entry before year t. 2. Next available after year t. 3. Compustat variable "sic" in year t.

For Compustat segments, Compustat reports up to three source dates per fiscal year. In subsequent filings, firms can re-organize their classification of segments, so I use the original source date when possible. An exception is if the sum of division sales and total sales from the main Compustat differ by more than 1%. Then, I iteratively go through subsequent source dates to check if one is within the threshold.

Table B1: **Data Definitions**. The default time subscript is t unless otherwise stated. Missing values for accounting variables are treated as zeros. Unless the definition links to an equation, data is from Compustat, Compustat Quarterly, Compustat Segments, CRSP, and Ken French's data library.

Uncertainty

Vol 5y	$\sqrt{252}\times$ std of excess stock returns from June $t-5$ to June $t-1$, daily.
Abnormal Vol 5y	$\sqrt{252}\times$ std of residuals from regression of excess stock return on Fama-French 5 factors from June $t-5$ to June $t-1$, daily.
Vol 5y de-levered	Vol 5y / (1+(dltt +dlc)/seq). When seq is missing, use ceq+pstk.
Vol 5y (monthly)	Vol 5y using monthly data instead of daily.
Cash Flow Vol 5y	$2\times$ std of quarterly sales from $t-4$ to t, scaled by at_{t-1} (20 obs).

Firm Investment & Controls

Investment rate	capx/(.5ppent+.5ppent$_{t-1}$), winsorized at [-.5,.5]
Net Hiring rate	(emp-emp$_{t-1}$)/(.5emp+.5emp$_{t-1}$)
R&D Investment rate	xrd/(.5K^K+.5K^K_{t-1}), K^K defined in Equation B1. Winsorized at .5.
Δ R&D expenditures	(xrd-xrd$_{t-1}$)/(.5xrd+.5xrd$_{t-1}$)
Δ Intangible Investment	(Intang-Intang$_{t-1}$)/(.5Intang+.5Intang$_{t-1}$), see below.
Tobin's Q	(at+csho∗prcc_f - ceq - txdb) / (.9∗at + .1∗(at+csho∗prcc_f - ceq - txdb)) Winsorized at 10.
Whited-Wu Index	.091∗(ib+dp)/at$_{t-1}$ - .062∗ $\mathbb{1}_{posdiv}$ + .021∗dltt/at$_{t-1}$ - .044∗log(at$_{t-1}$) + .1∗industry sales growth - .035∗sales growth, where $\mathbb{1}_{posdiv} = 1$ if dvc+dvp>0 and 0 otherwise
Intangible Investment	xrd + .3(xsga - xrd)
Payout	dvc + dvp + prstkc
Cash/Assets	che/at
Book Leverage	(dltt + dlc)/(dltt+dlc+seq). When seq is missing, use ceq+pstk.
Tangibility	ppegt/at$_{t-1}$
Operating Cash Flow	(oibdp - xint - txt)/at$_{t-1}$
Log Assets	Log(at)
Log Sales	Log(sale)
Return on Assets	ebit/at$_{t-1}$
Capex/Assets	capx/at$_{t-1}$
R&D/Assets	xrd/at$_{t-1}$
Sales Growth (firm)	sale/sale$_{t-1}$-1
Sales Growth (industry)	average sales growth of firms in the same 2-digit sich
Market-to-Book	(csho∗prcc_f)/ceq
Cumulative r^e	compounded daily excess stock returns from $t-5$ to June $t-1$.

Segments

Segment Net Hiring	(emp-emp$_{t-1}$)/(.5emp+.5emp$_{t-1}$)× sales/sale
Segment Sales Growth	sales/sales$_{t-1}$-1
Segment Log Sales	Log(sales)
Segment Log Assets	Log(ias)

Specificity

General (job postings)	See Equation 20.
relatedness (job postings)	See Equation 21.
General (patents)	See Equation B2.

B.2 Construction of knowledge capital

In the past few years, researchers have used the perpetual inventory method to construct a firm's stock of knowledge or intangible capital, which they have found to be consistent with practices at the Bureau of Economic Analysis. Peters and Taylor 2017, Belo et al. 2022 and Falato et al. 2022 are a few recent papers that take this approach.

The law of motion for knowledge capital is defined by

$$K_{t+1}^K = K_t^K(1-\delta^K)P_{t+1}^K/P_t^K + RD_{t+1} \tag{B1}$$

where K^K is the stock of knowledge capital, δ^K is the knowledge capital depreciation rate, P^K is the price index for R&D and RD_{t+1} is R&D expenditures at $t+1$. Moreover, the initial stock K_0^K is given by:

$$K_0^K = \frac{RD_0}{g^K + \delta^K - \pi^K(1-\delta^K)}$$

where π^K is the average growth rate of the price index for R&D and g^K is the sample median of the growth rate in R&D expenditures for a given industry. For many industries, the number of firms pooled across time is not well populated. As a result when there are fewer than 1000 total observations at the 2-digit SIC level from 1976-2022, I set growth rates to 0.0742, which is the median growth rate for industries exceeding this observation count.

For the price index, I use the series "Gross Private Domestic Investment: Fixed Investment: Nonresidential: Intellectual Property Products: Research and Development (chain-type price index)" provided by the Federal Reserve Economic Data (FRED) database.

Depreciation rates for 10 research intensive industries are provided in Li and Hall 2016[40]. Following Peters and Taylor 2017, I set $\delta^K = 0.15$ for unreported industries.

As discussed in Belo et al. 2022 footnote 12, write-off's are included in Compustat item XRD (R&D expenditure) whenever XRD_FN (XRD footnote) takes values "BW" or "BV". Despite being an expenditure, a write-off should not be included as part of capital stock, so I subtract the absolute value of item RDIP (In-Process R&D Expense) whenever BW or BV is coded.

Moreover, to get as much data as possible, I interpolate missing values for R&D.

i. First, I require that a firm has at least 50% non-missing observations for XRD. This is to ensure that I do not interpolate for firms who are not active in R&D, while filling in values for intermediate gaps, or the first or final few years that a firm is included in Compustat.

ii. When missing values are in the beginning, I use forward-looking 10 year averages of R&D as a percent of SG&A (item XSGA) where averages include missing observations as zeros.

[40] later versions of their paper, including Li and Hall 2020, omit the industry correspondence to SIC codes, so one should look for the one posted on the BEA's website, which they use as recommended depreciation rates.

iii. When missing values are at the end, I use backward-looking 10 year averages of R&D as a percent of SG&A. Moreover, I set interpolated R&D values for the last year the firm is in Compustat to missing.

iv. When missing values are in-between, I linearly interpolate.

B.3 Patent Matching

B.3.1 Similarity scores

I download all patents filed starting after 1975 from the United States Patent and Trademark Office (USPTO). I follow the data section and replication code in Kelly et al. 2021 closely when cleaning the patent data and computing cosine similarity scores, so I focus on where my process differs. First, I extend the data to patents granted by 2023 and update the dictionary accordingly.

Using their notation, for patent p, word w, I construct the "term-frequency-inverse-document-frequency" (TFIDF) as

$$TFIDF_{pw} \doteq TF_{pw} \times IDF_w$$

where

$$TF_{pw} = \frac{c_{pw}}{\sum_k c_{pk}}$$

measures the relative importance of term w for patent p, c_{pw} is the number of times word w is used in patent p, and $\sum_k c_{pk}$ is the total number of words used in the patent. Moreover,

$$IDF_w \doteq \log\left(\frac{\#\text{documents in sample within 10 years}}{\#\text{documents that include term } w \text{ within 10 years}}\right)$$

To quote Kelly et al. 2021 on what IDF measures, "*IDF* measures the informativeness of term w by underweighting common words that appear in many documents ... a high value of $TFIDF_{pw}$ indicates that term w appears relatively frequently in document p but does not appear in most other documents, thus conveying that word w is especially representative of document p's semantic content." In my modification, I restrict the set of patents to patents filed within 10 years. The precise number, 10, is arbitrary, but included to limit the sensitivity of a word's informativeness by the spillover of new inventions into new industries; one concern is that a very specific patent in an early time period may no longer be specific after decades of knowledge diffusion.

For each patent, the vector V_p is a collection of $TFIDF_{pw}$ normalized to have unit length,

$$V_p = \frac{TFIDF_{pw}}{||TFIDF_{pw}||}$$

The cosine similarity score between patent p and patent q is

$$\rho_{p,q} = V_p \cdot V_q$$

$\rho_{p,q} \in [0,1]$ where complete overlap is $\rho_{p,q} = 1$ and no overlap is $\rho_{p,q} = 0$. Finally, as in Kelly et al. 2021, I set pairwise-similarity scores below 5% to zero in order to reduce the size and computational burden.

B.3.2 Patent-industry mapping

I match patents to firms to identify its corresponding industry[41]. There are several existing papers that have mapped patents to public firms in Compustat and CRSP, such as Autor et al. 2020, Kogan et al. 2017, Arora et al. 2021, the NBER patent database, among many others. I take the union of these datasets and make corrections when the data disagree using the original patent filing.

In many cases, a public firm in Compustat is assigned ownership of a patent that was originally filed by a different company that the public firm subsequently acquired. This change in ownership must be reported to the USPTO, and as a result, the google patents webpage for that patent keeps a historical record of changes in ownership. Taking advantage of this feature, I match patents to both public and private firms by mapping the target-acquirer data from the SDC Platinum M&A database to historical ownership changes. If for patent p I see a change in ownership from x to y, and I also see x is acquired by y in around the same time horizon (usually within 1-3 years), I can then infer the associated industry for patent p because SDC lists both the target and acquirer's industries. This process requires iterative fuzzy matching on the target's name and extensive validation checks. However, it is a necessary step because the currently available patent-firm mappings are single snapshots in time, so one runs the risk of erroneously attributing ownership of a patent to a firm before it was acquired and assigning the incorrect industry for that patent. As a result I exclude patents from the existing patent-firm keys in which the filing date is before the date of announcement of the acquisition.

Because I require a patent-firm-industry mapping, the sample size is more than halved from 7.9 million patents to 3.1 million in my data sample. When computing similarity scores between patents, I restrict to patents filed within ± 5 years of each other to reduce the computational burden, as well as possible diffusion of knowledge from one industry to the next. Once a patent is matched to a firm, I first assign the patent the union of the firm's associated industries via Compustat and Compustat segments data. One issue is that many firms report multiple segments. Moreover, whether a patent should be assigned to multiple industries because its parent firm operates in multiple industries is ambiguous. Therefore, I construct the measure using both multiple and single industry assignment. For single assignment, suppose that a patent is filed by a multisegment firm in A, B. I compare its similarity score to all patents filed by single-segment firms in A, then to all patents filed by single-segment firms in B, then assign the industry with the highest textual similarity as the relevant industry. In subsection B.3.3 below, I describe in more detail about this procedure.

Moreover, I categorize industries based on both 2-digit SIC codes and Fama-French 49 industry groups.

[41] It is not ideal to use the patent classification system because by construction then, patents will be more textually similar to patents within its own classification with little variation across patents.

For each patent p, I compute its General score as its average cosine similarity to all other patents that are filed outside of its own industry, or

$$\text{General}_p = \frac{1}{|\{q : q \in \mathscr{I} \backslash \{j\}\}|} \sum_{q \in \mathscr{I} \backslash \{j\}} \rho_{p,q} \tag{B2}$$

where $\mathscr{I}$ is the set of 2-digit SIC codes and j is the industry to which it is classified in.

B.3.3 An Example of Single Industry Assignment

Table B2: This table shows the intermediate step in the patent-industry mapping for patents filed by multi-segment firms. For each firm, the column "sic-2" shows the firm's reported industry segments in Compustat segments for the corresponding year the patent was originally filed. I then compute textual analysis on a given patent p listed under "Patent No." against all patents filed by single segment firms in each reported industry within a 10 year window of the filing date. The column "# > 5%" reports the number of patents from each industry that have a similarity score greater than 5% with patent p; the column "Total #" reports the total number of patents filed in these industries by single segment firms during this time. The column "Pct > 5%" is the fraction of patents from that industry that exceed the 5% threshold and the column "Sim" is the average similarity score between p and these patents (out of 1,000). Finally, the indicator variables take the value of 1 if patent p takes the highest values for "# > 5%", "Pct > 5%", "Sim" within its reported industries.

Firm	Patent No.	sic-2	# > 5%	Total #	Pct > 5%	Sim	$\mathbb{1}_{\text{count}}$	$\mathbb{1}_{\text{pct}}$	$\mathbb{1}_{\text{sim}}$
Hitachi	7185048	17	1	40	0.025	NaN	-	-	-
	7185048	28	57	65,568	0.001	72.00	-	-	-
	7185048	35	2,651	68,719	0.039	103.29	1	1	1
	7185048	36	1,780	180,853	0.010	87.52	-	-	-
	7185048	61	18	706	0.025	102.29	-	-	-
P&G	9404070	26	393	2,250	0.175	83.03	-	-	-
	9404070	28	23,767	34,528	0.688	100.82	1	1	1
	9404070	36	1,338	179,709	0.007	76.09	-	-	-
J&J	11187920	28	10,961	26,327	0.416	93.24	1	1	-
	11187920	38	1,657	29,950	0.055	126.91	-	-	1

For more than half of the patents filed by multi-segment firms, requiring that all three indicator variables as defined in Table B2 take the value of 1 is a reliable method to assign its corresponding industry[42]. In the following table, consider patent number 9404070 filed by Procter and Gamble Co. The title of this invention is "Compositions and methods for surface treatment with lipases" and the first sentence of the description is "This invention relates to compositions comprising lipase enzymes and bleaching agents, as well as methods of making and using such compositions which are preferably fabric and home care products". This is an invention for detergent. Out of P&G's three reported industry segments at the

[42]In a random sample of 100 patents that meet this criteria, all 100 patents matched to its correct industry.

time of the filing, this patent has the highest number of matches (exceeds 5% similarity) with patents filed by single industry firms in the 2-digit SIC code 28 for "Chemical & Allied Products", which includes "Soap, Detergents, Cleaning Preparations..." (SIC code 2840). Out of the $34,528$ patents filed by single-segment firms in the 2-digit SIC code 28 over this span, almost 70% of patents have similarity scores greater than 5%, and the average similarity per matched patent (100.82) is highest relative to its other reported industries. Based on the description of this patent, this patent is unlikely developed by P&G's "Paper & Allied Products" (26) nor "Electronic & Other Electric Equipment" (36) divisions, and the indicator variables do a good job have capturing this inference.

A slightly more involved example is 7185048 filed by Hitachi LTD. The title of this invention is "Backup processing method" and the first sentence of the abstract is "A backup processing method for backing up data to be used by a data-processing computer system comprises selecting resources in a usable state from a plurality of resources necessary for the data to be used by the data-processing computer system..." Under my decision rule, this patent is categorized under "Industrial Machinery & Equipment", which includes "Computer Storage Devices" (SIC code 3572). This patent is not "Special Trade Contractors" (17), "Chemicals & Allied Products", "Electronics & Other Electric Equipment" (36, which excludes computers), nor "Nondepository Institutions" (61). However, we can see in the above table the statistics in the industry can be misleading if the number of patents filed in an industry is low. In many cases, there is high overlap with patents filed by financial institutions, whose patents are mostly administrative and management related in nature. Excluding the financing division of these multi-segment firms quickly resolves this problem.

When one industry does not score 1 for all three indicator variables, requiring that two of the three take values of 1 is the next-best method. In the above table, I present a case in which the decision rule leads to an error, patent 11187920 filed by Johnson & Johnson Vision Care Inc. This patent is titled "Increased stiffness center optic in soft contact lenses for astigmatism correction". Strictly speaking, Johnson & Johnson lists disposable contact lenses under its "Medical Devices" segment in its 10-k and is consistent with the SIC code 3851 for "Ophthalmic Goods". As a result, this patent has the highest average similarity of matched patents, 126.91, to patents filed by firms in the 2-digit SIC code 38. However, pages and pages of this patent include lists of chemical compounds, which leads to a high overlap with patents filed in "Chemical & Allied Products" which include both J&J's pharmaceutical and consumer businesses. One may argue that disposable contact lenses appropriately falls under its consumer segment, as opposed to its medical devices business which sells laboratory, precision and surgical tools. However, relying on $\mathbb{1}_{\text{sim}}$ is less reliable for firms whose businesses are not as related as Johnson & Johnson's. In random samples, the error rate of this decision is below 10%; nevertheless, this problem is substantially improved when switching to Fama-French 49 industries, which correctly classifies this patent.

C Appendix: Extensions

C.1 Generalist and specialist firms

Table C1: **Average $\mathfrak{z}/\mathfrak{k}$ of projects across all sample paths.** The below table shows the average ratio $\mathfrak{z}/\mathfrak{k}$ (as opposed to the ratio $\mathfrak{k}/\mathfrak{z}$ in the main body of the paper) of all investment projects in simulated data, then compares the average characteristics of projects that are adopted by a firm when varying the firm's initial distribution of general and specific capital. I refer to these firms as balanced ($z^i = z^j = k^i = k^j = 10$), a generalist ($z^i = z^j = 5$ and $k^i = k^j = 15$), and a specialist ($z^i = z^j = 15$ and $k^i = k^j = 5$).

	$\sigma = 0.05$			$\sigma = 0.12$		
Average $\mathfrak{z}/\mathfrak{k}$, all projects	1.65			1.67		
	Balanced	Generalist	Specialist	Balanced	Generalist	Specialist
Average $\mathfrak{z}/\mathfrak{k}$, adopted projects	1.20	1.42	1.32	1.73	1.75	1.66

I repeat the simulation exercise as before while varying the firm's initial characteristics. In each sample path, there is a balanced firm ($z^i = z^j = k^i = k^j = 10$), a generalist ($z^i = z^j = 5$ and $k^i = k^j = 15$), and a specialist ($z^i = z^j = 15$ and $k^i = k^j = 5$). To keep as much control as possible, the three firms are exposed to the same randomness within a sample path: in addition to the state variable s_t, the arrival process for investment projects and draws for the project composition are identical for each firm. In other words, projects arrive at the same time, are of the same make-up, and I record which firms adopt it and which ones do not. I simulate 5,000 sample paths when $\sigma = 0.05$ and another 5,000 when $\sigma = 0.12$.

Table C1 shows summary statistics for the ratio $\mathfrak{z}/\mathfrak{k}$ of investment projects that are drawn, pooled across all sample paths for a given σ. The average $\mathfrak{z}/\mathfrak{k}$ of all projects is about 1.65 though $\mathfrak{z}$ and $\mathfrak{k}$ are both drawn from uniform distributions on $[.05, .95]$; this is just an artifact of the ratio having a lower bound at zero and hence, right-skewed. In the left panel when $\sigma = 0.05$, we can see how the average $\mathfrak{z}/\mathfrak{k}$ of projects that are adopted by the balanced firm, indicated by the column "Balanced" drops to 1.20. This is expected, evidenced by the highly specialized projects that are skipped in Figure 4. However on the right panel, we see how the average $\mathfrak{z}/\mathfrak{k}$ increases to 1.73 when $\sigma = 0.12$, higher than the unconditional average 1.67. Indeed, there appears to be differences between a balanced firm, a generalist and a specialist, but the first-order effect is the level of uncertainty in the underlying markets.

C.2 The one-product firm

In this section, I study a single segment firm that has an expansion option to enter a second market. To streamline interpretability, I consider the simplified case when $\theta = 2, \alpha = 0.5, \varepsilon \to 0$, but formulas with CES production are nearly identical except for expressions for the complementarities of an investment project.

Suppose there is a single segment firm that operates in industry i with production technology $z^i k^i$. With no investment projects, firm value is given by the Gordon-growth formula $z^i k^i/\delta^i$. With a perpetual option to adopt a single project j that gives the firm $z^j k^j$ in addition to the option to redeploy general capital, firm value at value-matching is:

$$\frac{z^i k^i}{\delta^i} + \mathbb{C}_6 e^{\phi^+ s} = \frac{z^i k^i}{\delta^i} + \frac{z^j k^j}{\delta^j} e^s + \mathbb{C}_1 e^{\phi^+ s} + \mathbb{C}_2 e^{\phi^- s} - f$$

and we return to the familiar setup of the multiproduct firm. The cost of the acquisition f can be interpreted as a one time post-merger integration expense of learning to operate multiple product markets, and we can apply the optimality condition to solve for the boundary $\widehat{s}$ as before.

With multiple project arrivals in i and j, suppose the first acquisition in j requires an entry setup cost f, but subsequent acquisitions are priced at fair value $\varpi^j s$ as before. Then, the optimal exercise boundary for the first project j adoption satisfies

$$\begin{aligned}
0 &= (1-\phi_1^+)\overbrace{\left[\frac{z^j k^j}{\delta^j} + \frac{\lambda^j}{\delta^j + \lambda^i + \lambda^j}\frac{\tilde{z}^j k^j}{\delta^j}\right]}^{\doteq \varphi^j} \times e^s + \phi_1^+ f \\
&+ (\phi_0^+ - \phi_1^+)\left[\frac{\lambda^i}{\lambda^i+\lambda^j}\mathbb{D}_1^i(k^i + \tilde{\mathscr{k}}) + \frac{\lambda^j}{\lambda^i+\lambda^j}\left[\mathbb{D}_1^j(k^i) - \mathbb{C}_1^j(k^i)\right]\right] \times e^{\phi_0^+ s} \\
&+ (\phi_0^- - \phi_1^+)\left[\frac{\lambda^i}{\lambda^i+\lambda^j}\mathbb{D}_2^i(k^j) + \frac{\lambda^j}{\lambda^i+\lambda^j}\left[\mathbb{D}_2^j(k^j + \tilde{\mathscr{k}}) - \mathbb{C}_2^j(k^j)\right]\right] \times e^{\phi_0^- s} \\
&+ (\phi_1^- - \phi_1^+) \times e^{\phi_1^- s}
\end{aligned}$$

One way to simplify the above expression is first by assuming that the firm needs to accumulate some minimum capital level $\underline{k}^j$ before it can use its option to reallocate assets. Because z^i is much greater than z^j for the single-segment firm, this assumption prevents the firm from acquiring $z^j k^j$ to immediately reallocate k^j to division i given $z^i k^j > z^j k^j$. Assuming that the initial project $k^j < \underline{k}^j$, we have that the acquisition occurs the first time $s \geq \check{s}$, where

$$\check{s} = \left(\frac{\phi_1^+}{\phi_1^+ - 1}\right)\frac{f}{\varphi^j}$$

For the parameters I use in this paper, if $f = z^j k^j/\delta^j$, the fair value of the production technology, $\exp \check{s} \approx 1$, meaning the firm expands into j as soon as $A^j > A^i$. If f is 1.5 times the fair value, $\exp \check{s} \approx 1.6$, implying that industry j must be doing well relative to i before the first project in j is adopted (since $s = \log(A^j/A^i)$). After the first project j, the cost reverts to $\varpi^j e^s$, and until the firm accumulates a sufficient amount of capital $k^j \geq \underline{k}^j$, no stopping time exists for future projects in j, meaning that once the firm diversifies, it quickly grows until it accumulates $\underline{k}^j$. Once capital in place exceeds $\underline{k}^j$, projects are evaluated in context of how it also affects the reallocation channel, returning to the multiproduct firm's problem as before.

One can follow the same steps to derive expressions for the project adoption threshold for

Table C2: **Average $\mathfrak{z}/\mathfrak{k}$ of projects across all sample paths.** The below table shows the average ratio $\mathfrak{z}/\mathfrak{k}$ of all investment projects in simulated data, then compares the average characteristics of projects that are adopted by a firm depending on the firm's choice of organizational form.

	$\sigma = 0.05$			$\sigma = 0.10$		
Average $\mathfrak{z}/\mathfrak{k}$, all projects	1.65			1.67		
	Firm ij	Firm i	Firm j	Firm ij	Firm i	Firm j
Average $\mathfrak{z}/\mathfrak{k}$, adopted projects	1.12	1.54	1.55	1.65	1.66	1.66
As single market		1.65	1.65		1.65	1.65
As expansion option		1.05	1.05		1.13	1.14

a project in i, and solve for the near identical case when the firm starts in product market j and has an expansion option into i.

In simulations, values of f are chosen with CES production technology that are 1.5 times the fair value of the project, which of course changes for different levels of σ. Regardless, expansion occurs in approximately 50% of the same paths.

To establish a counterfactual, by comparing the average characteristics of investment projects that we do observe to that of firms who not share the incentive to maximize over the reallocation channel−namely, one-product firms. Furthermore, the firm has a perpetual option to expand into a secondary market.

I repeat the simulation exercise as before except I use $\sigma = 0.10$ instead of $\sigma = 0.12$ for additional comparisons. In each sample path, there is a two-product firm, a firm that starts in product market i with a perpetual option to expand into product market j, and a firm in j that can expand into i. For brevity, I refer to these firms as "Firm ij", "Firm i" and "Firm j" respectively. Again, to keep as much control as possible, the three firms are exposed to the same randomness within a sample path.

In contrast to Table C1, the average $\mathfrak{z}/\mathfrak{k}$ of a one-product firm only drops, albeit slightly, once the expansion option is exercised[43]. Furthermore, we confirm the effects of higher volatility. When $\sigma = 0.12$, we cannot immediately observe any differences among the three firms. Finally, the last row in Table C2 shows the average $\mathfrak{z}/\mathfrak{k}$ of the first project that is adopted when the firm expands from a one-product firm to a two-product firm.

Because I estimate values of $\mathfrak{z}/\mathfrak{k}$ for targets of acquisitions, it is instructive to compare equivalent regressions between simulated and actual data. In Table C3, I estimate

$$\mathfrak{z}/\mathfrak{k}_{nt} = \beta_0 + \beta_1 \text{Two Market}_{nt} + \sum_d \beta_d \text{Control}_{d,nt} + u_{nt} \tag{C1}$$

[43]Given how the cost of expansion is setup in this section, expansion occurs in roughly half of the sample paths. Moreover, when the option is exercised, it occurs roughly 7 years into the 10 year path.

and

$$\mathfrak{z}/\mathfrak{k}_{nt} = \beta_0 + \beta_1 \text{Two Market}_{nt} + \beta_2 \text{Two Market}_{nt} \times \sigma_n + \sum_d \beta_d \text{Control}_{d,nt} + u_{nt} \qquad \text{(C2)}$$

for $n = 1...40000,\ t = 1...10 \times 12$, where the dependent variable $\mathfrak{z}/\mathfrak{k}_{nt}$ is ratio of specific-to-general production technology of an investment project that is adopted at time t in sample path n. The main independent variable is the dummy variable Two Market$_{nt}$ which takes the value of 1 if the firm, at the time the firm invests in the project, is a two-product firm. In Equation C2, the interaction term Two Market$_{nt} \times \sigma_n$ is the dummy variable Two Market, defined as before, multiplied by the volatility of the underlying stochastic processes. Because Table C3 is a pooled time-series, $\sigma = \sigma^i = \sigma^j = 0.05$ for 20,000 sample paths and $\sigma = 0.10$ for the remaining 20,000. Equation C1 tests how firm boundaries, which introduce the reallocation channel in this model, affect the average characteristics of projects that are adopted, and Equation C2 tests whether the effect is declining in volatility.

The first column of Table C3 estimates the regression without any controls, so the coefficient $\beta_0 = 1.65$ corresponds to the average ratio of all projects that are adopted. The coefficient $\beta_1 = -0.18$ is the average effect conditional on the firm, at the time of investment, is a two-product firm. The average effect is muted because it pools together $\sigma = 0.05$ and $\sigma = 0.10$ results. When including the volatility interaction in column 4, we see the magnitude drop sharply to $\beta_1 = 0 - .77$, consistent with Table C2, and the coefficient of 0.08 on $TwoMarket \times \sigma$ says that a firm with $\sigma = 0.10$ effectively offsets $\beta_1 = 0 - .77$ with $\beta_4 = 0.80$, so the average effect only holds when the two-product firm operates in a low volatile environment. Finally, column 6 adds sample path fixed effects, which control for the randomness within a given sample path and is the appropriate control in the regression.

Overall, the regression table shows that when comparing the choice of organizational form on the characteristics of investment projects, the multiproduct firm is decidedly a generalist in comparison, but again, only when volatility is low.

Table C3: **Characteristics of adopted projects from simulated sample paths.** The below table is a regression on the pooled

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Two Market	-0.18***	-0.15***	-0.15***	-0.76***	-0.77***	-0.50***	-0.33***	-0.29***
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)
Project Size		-0.79***	-0.78***	-0.78***	-0.77***	-0.74***		-0.74***
		(0.00)	(0.00)	(0.00)	(0.00)	(0.00)		(0.00)
σ			0.05***	0.00	0.00	0.01***	0.05***	0.04***
			(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Two Market $\times$ σ				0.08***	0.08***	0.05***	0.03***	0.03***
				(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Expansion					-0.14***	-0.09***	-0.30***	-0.09***
					(0.01)	(0.00)	(0.00)	(0.00)
Constant	1.64***	2.48***	2.08***	2.46***	2.46***	2.32***	1.17***	1.99***
	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Time $\times$ Simulation FE	no	no	no	no	no	yes	yes	yes
Firm FE	no	no	no	no	no	no	yes	yes
R^2	0.004	0.028	0.032	0.035	0.035	0.620	0.612	0.629
Observations	4,111,886	4,111,886	4,111,886	4,111,886	4,111,886	4,111,886	4,111,886	1,965,214

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

C.3 Stochastic Volatility

In this section, I introduce a two-state Markov regime-switching process. I assume there are two volatility regimes $\nu \in \{L, H\}$ and use $\eta^L \mathbf{d}t, \eta^H \mathbf{d}t$ to denote the risk-neutral probabilities of a transition. Moreover, I assume that outstanding investment projects still remain after a volatility regime change, though they still expire at the arrival of the next project. A H volatility regime can only be followed by a L regime and vice-versa, while there are no restrictions on the sequence of project arrivals. Changing notation, let V^H denote the value function in volatility regime H in which there are no projects outstanding (the firm is waiting for the next arrival). Let $V^{H,i}$ denote the value function in regime H in which the firm currently has the option to invest in an i project. Furthermore, let $V_s^{H,i}, V_{ss}^{H,i}$ denote the first and second partial derivatives of the value function with respect to the state variable s. Finally, let $\zeta \doteq [\alpha(z^i)^\epsilon + (1-\alpha)(k^i)\epsilon]^{\theta/\epsilon} + [\alpha(z^j)^\epsilon + (1-\alpha)(k^j)\epsilon]^{\theta/\epsilon} e^s$. Following this pattern, firm value is characterized by the system of ODEs:

$$
\begin{aligned}
\tilde{r^H} V^H &= \zeta + \tilde{\mu^H} V_s^H + \frac{1}{2}(\tilde{\sigma^H})^2 V_{ss}^H + \eta^L(V^L - V^H) + \lambda^i(V^{H,i} - V^H) + \lambda^j(V^{H,j} - V^H) \\
\tilde{r^H} V^{H,i} &= \zeta + \tilde{\mu^H} V_s^{H,i} + \frac{1}{2}(\tilde{\sigma^H})^2 V_{ss}^{H,i} + \eta^L(V^{L,i} - V^{H,i}) + \lambda^j(V^{H,j} - V^{H,i}) \\
\tilde{r^H} V^{H,j} &= \zeta + \tilde{\mu^H} V_s^{H,j} + \frac{1}{2}(\tilde{\sigma^H})^2 V_{ss}^{H,j} + \eta^L(V^{L,j} - V^{H,j}) + \lambda^i(V^{H,i} - V^{H,j}) \\
\tilde{r^L} V^L &= \zeta + \tilde{\mu^L} V_s^L + \frac{1}{2}(\tilde{\sigma^L})^2 V_{ss}^L + \eta^H(V^H - V^L) + \lambda^i(V^{L,i} - V^L) + \lambda^j(V^{L,j} - V^L) \\
\tilde{r^L} V^{L,i} &= \zeta + \tilde{\mu^L} V_s^{L,i} + \frac{1}{2}(\tilde{\sigma^L})^2 V_{ss}^{L,i} + \eta^H(V^{H,i} - V^{L,i}) + \lambda^j(V^{L,j} - V^{L,i}) \\
\tilde{r^L} V^{L,j} &= \zeta + \tilde{\mu^L} V_s^{L,j} + \frac{1}{2}(\tilde{\sigma^L})^2 V_{ss}^{L,j} + \eta^H(V^{H,j} - V^{H,j}) + \lambda^i(V^{L,i} - V^{L,j}) \qquad \text{(C3)}
\end{aligned}
$$

where $\tilde{r^\nu}, \tilde{\mu^\nu}, \tilde{\sigma^\nu}$ follow Equation 5 using the corresponding regime's parameters after an application of Girsanov's theorem for semi-martingales.[44] Since this is a system of second-order linear ODEs, it is convenient to reduce the above to a system of first-order linear ODEs. Define

$$
\xi^H \doteq V_s^H \quad ; \quad \xi^{H,i} \doteq V_s^{H,i} \quad ; \quad \xi^{H,j} \doteq V_s^{H,j} \quad ; \quad \xi^L \doteq V_s^L \quad ; \quad \xi^{L,i} \doteq V_s^{L,i} \quad ; \quad \xi^{L,j} \doteq V_s^{L,j}
$$

Collecting in matrix form,

$$
\nabla \mathbf{V} = \mathbf{A}\mathbf{V} + \mathbf{B}
$$

[44] Since I assume an exogenous SDF in this paper, I require additional assumptions such as a price of risk γ attached to volatility regime changes in which γ satisfies no-arbitrage and translates risk-neutral transition probabilities to physical probabilities. This can be done by deriving the equivalent martingale measure.

where

$$\mathbf{V} = \begin{bmatrix} V^H \\ V^{H,i} \\ V^{H,j} \\ V^L \\ V^{L,i} \\ V^{L,j} \\ \xi^H \\ \xi^{H,i} \\ \xi^{H,j} \\ \xi^L \\ \xi^{L,i} \\ \xi^{L,j} \end{bmatrix} \quad \text{and} \quad \mathbf{A} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{A}_4 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} \zeta \\ \zeta \\ \zeta \\ \zeta \\ \zeta \\ \zeta \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3, \mathbf{A}_4$ are each 6×6. $\mathbf{A}_1$ is a matrix of zeros, $\mathbf{A}_2$ is an identity matrix,

$$\mathbf{A}_3 = \begin{bmatrix} -\frac{r^{\tilde{H}}+\eta^L+\lambda^i+\lambda^j}{\sigma^{\tilde{H}}} & \frac{\lambda^i}{\sigma^{\tilde{H}}} & \frac{\lambda^j}{\sigma^{\tilde{H}}} & \frac{\eta^L}{\sigma^{\tilde{H}}} & 0 & 0 \\ \frac{\lambda^i}{\sigma^{\tilde{H}}} & -\frac{r^{\tilde{H}}+\eta^L+\lambda^i+\lambda^j}{\sigma^{\tilde{H}}} & \frac{\lambda^j}{\sigma^{\tilde{H}}} & 0 & \frac{\eta^L}{\sigma^{\tilde{H}}} & 0 \\ \frac{\lambda^j}{\sigma^{\tilde{H}}} & \frac{\lambda^i}{\sigma^{\tilde{H}}} & -\frac{r^{\tilde{H}}+\eta^L+\lambda^i+\lambda^j}{\sigma^{\tilde{H}}} & 0 & 0 & \frac{\eta^L}{\sigma^{\tilde{H}}} \\ \frac{\eta^H}{\sigma^{\tilde{L}}} & 0 & 0 & -\frac{r^{\tilde{L}}+\eta^H+\lambda^i+\lambda^j}{\sigma^{\tilde{L}}} & \frac{\lambda^i}{\sigma^{\tilde{L}}} & \frac{\lambda^j}{\sigma^{\tilde{L}}} \\ 0 & \frac{\eta^H}{\sigma^{\tilde{L}}} & 0 & \frac{\lambda^i}{\sigma^{\tilde{L}}} & -\frac{r^{\tilde{L}}+\eta^H+\lambda^i+\lambda^j}{\sigma^{\tilde{L}}} & \frac{\lambda^j}{\sigma^{\tilde{L}}} \\ 0 & 0 & \frac{\eta^H}{\sigma^{\tilde{L}}} & \frac{\lambda^j}{\sigma^{\tilde{L}}} & \frac{\lambda^i}{\sigma^{\tilde{L}}} & -\frac{r^{\tilde{L}}+\eta^H+\lambda^i+\lambda^j}{\sigma^{\tilde{L}}} \end{bmatrix}$$

and

$$\mathbf{A}_4 = \begin{bmatrix} -\frac{\mu^{\tilde{H}}}{\sigma^{\tilde{H}}} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\mu^{\tilde{H}}}{\sigma^{\tilde{H}}} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mu^{\tilde{H}}}{\sigma^{\tilde{H}}} & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{\mu^{\tilde{L}}}{\sigma^{\tilde{L}}} & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{\mu^{\tilde{L}}}{\sigma^{\tilde{L}}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\mu^{\tilde{L}}}{\sigma^{\tilde{L}}} \end{bmatrix}$$

The general solution is

$$\mathbf{V} = \sum_{n}^{12} c_n \mathbf{v}_n e^{\omega_n s} + \mathbf{b}$$

where for each eigenvalue ω_n, $\mathbf{v}_n$ is the corresponding eigenvector. Moreover, the integration constants $c_n, n = 1, ..., 12$ are determined by writing the boundary conditions in Equations A19 and A20 from Appendix section A.4 for each volatility regime, then passing through to the newly defined variables in the system of first-order equations. For example,

$\partial^2 V^H/\partial k^i \partial s = 0$ passes over to ξ^H as $\partial \xi^H/\partial k^i = 0$.

For the column matrix $\mathbf{b}$, $\mathbf{b} = \begin{bmatrix}\mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3\end{bmatrix}^\top$, where $\mathbf{b}_1, \mathbf{b}_2$ are each 3×1 and $\mathbf{b}_3$ is a 6×1 column of zeros. $\mathbf{b}_2$ can be written by reversing the H's and L's from $\mathbf{b}_1$, where $\mathbf{b}_1 = \begin{bmatrix}b_1 & b_1 & b_1\end{bmatrix}^\top$ and

$$
\begin{aligned}
b_1 = & \frac{\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{H,i} + \lambda^i + \lambda^j + \eta^L} + \frac{\lambda^i}{\delta^{H,i} + \lambda^i + \lambda^j + \eta^L} \frac{\left[\alpha(z^i + \tilde{z}^i)^\varepsilon + (1-\alpha)(k^i + \tilde{k}^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{H,i} + \lambda^i + \lambda^j + \eta^L} \\
& + \frac{\lambda^j}{\delta^{H,i} + \lambda^i + \lambda^j + \eta^L} \frac{\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{H,i} + \lambda^i + \lambda^j + \eta^L} + \frac{\left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{L,i} + \lambda^i + \lambda^j + \eta^H} \\
& + \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{H,j} + \lambda^i + \lambda^j + \eta^L} e^s + \frac{\lambda^j}{\delta^{H,j} + \lambda^i + \lambda^j + \eta^L} \frac{\left[\alpha(z^j + \tilde{z}^j)^\varepsilon + (1-\alpha)(k^j + \tilde{k}^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{H,j} + \lambda^i + \lambda^j + \eta^L} e^s \\
& + \frac{\lambda^i}{\delta^{H,j} + \lambda^i + \lambda^j + \eta^L} \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{H,j} + \lambda^i + \lambda^j + \eta^L} e^s + \frac{\left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}}}{\delta^{L,j} + \lambda^i + \lambda^j + \eta^H} e^s \\
& - \frac{\lambda^i}{\delta^{H,i} + \lambda^i + \lambda^j + \eta^L} \varpi^i - \frac{\lambda^j}{\delta^{H,j} + \lambda^i + \lambda^j + \eta^L} \varpi^j e^s
\end{aligned}
$$

As before, the integration constants can be solved semi-explicitly while the investment and reallocation free boundaries must be determined numerically following the steps in Appendix section A.4.

When η^H and η^L are calibrated such that periods of high volatility are short-lasting ($\eta^L >> \eta^H$), the model can then explain both the negative coefficient for short-horizons and the positive coefficient for medium to long horizons. When $\eta^H = \eta^L$, then investment rates approximate the average of σ^H and σ^L, leading only to the positive relationship between investment and firm-level uncertainty.

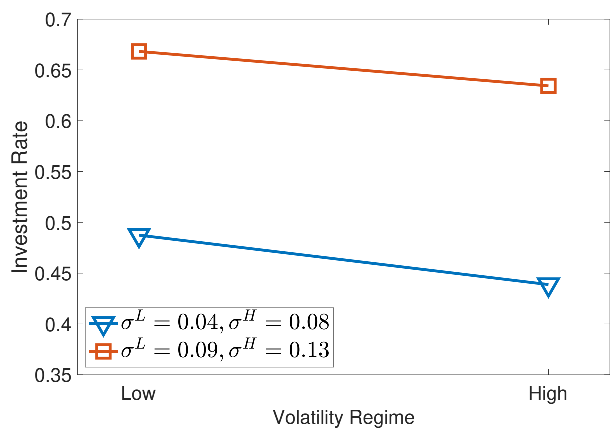


Figure C1: **Average investment rate in regimes L and H.**

C.4 Separate Factor Markets

Suppose the firm can incrementally add each type of capital: k^i, z^i, k^j, z^j (general and specific inputs in product markets i and j). For convenience, I assume no depreciation so the law of motion for capital stock follows:

$$\begin{aligned}
\mathbf{d}k^i &= I_{k^i}\mathbf{d}t \\
\mathbf{d}k^j &= I_{k^j}\mathbf{d}t \\
\mathbf{d}z^i &= I_{z^i}\mathbf{d}t \\
\mathbf{d}z^j &= I_{z^j}\mathbf{d}t
\end{aligned}$$

For each input, investment incurs a quadratic cost as is standard in the neoclassical investment literature. For example, the cost of investing I_{k^i} is: $\frac{1}{2\varphi}{I_{k^i}}^2$, where φ is a common parameter across the four types of inputs. The HJB equation for the firm's problem can be written,

$$\begin{aligned}
\tilde{r}H(s,k^i,k^j) = \max_{I_{k^i},I_{z^i},I_{k^j},I_{z^j}} \quad & \left[\alpha(z^i)^\varepsilon + (1-\alpha)(k^i)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} + \left[\alpha(z^j)^\varepsilon + (1-\alpha)(k^j)^\varepsilon\right]^{\frac{\theta}{\varepsilon}} e^s \\
+ \tilde{\mu}\frac{\partial H(s,k^i,k^j)}{\partial s} + \frac{1}{2}(\tilde{\sigma})^2\frac{\partial^2 H(s,k^i,k^j)}{\partial s^2} &+ I_{k^i}\frac{\partial H}{\partial k^i} - \frac{1}{2\varphi}{I_{k^i}}^2 + I_{z^i}\frac{\partial H}{\partial z^i} - \frac{1}{2\varphi}{I_{z^i}}^2 \\
+ I_{k^j}\frac{\partial H}{\partial k^j} - \frac{1}{2\varphi}{I_{k^j}}^2 + I_{z^j}\frac{\partial H}{\partial z^j} - \frac{1}{2\varphi}{I_{z^j}}^2 &
\end{aligned}$$

with boundary conditions:

$$\begin{aligned}
\frac{\partial H(\bar{s},k^i,k^j)}{\partial k^j} - \frac{\partial H(\bar{s},k^i,k^j)}{\partial k^i} = c \quad &; \quad \frac{\partial H(\underline{s},k^i,k^j)}{\partial k^i} - \frac{\partial H(\underline{s},k^i,k^j)}{\partial k^j} = ce^{\underline{s}} \\
\frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^j \partial s} - \frac{\partial^2 H(\bar{s},k^i,k^j)}{\partial k^i \partial s} = 0 \quad &; \quad \frac{\partial^2 H(\underline{s},k^i,k^j)}{\partial k^i \partial s} - \frac{\partial^2 H(\underline{s},k^i,k^j)}{\partial k^j \partial s} = ce^{\underline{s}}
\end{aligned}$$

Taking first-order conditions, optimal investment in each input is:

$${I_{k^i}}^* = \varphi\frac{\partial H}{\partial k^i} \quad ; \quad {I_{z^i}}^* = \varphi\frac{\partial H}{\partial z^i} \quad ; \quad {I_{k^j}}^* = \varphi\frac{\partial H}{\partial k^j} \quad ; \quad {I_{z^j}}^* = \varphi\frac{\partial H}{\partial z^j}$$

Substituting optimal investment into the boundary conditions for reallocation, we have the value-matching condition to reallocate from i to j,

$$\frac{1}{\varphi}\left({I_{k^j}}^* - {I_{k^i}}^*\right) = c$$

In other words, the value-matching condition equates the cost of adjusting capital externally against the cost of adjusting capital internally.

In Figure C2, I plot the average investment rate for each type of capital input, averaged

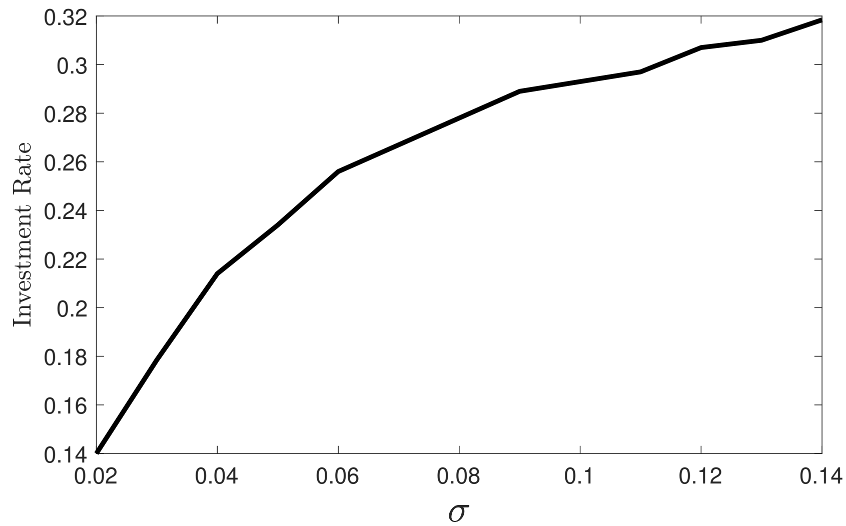
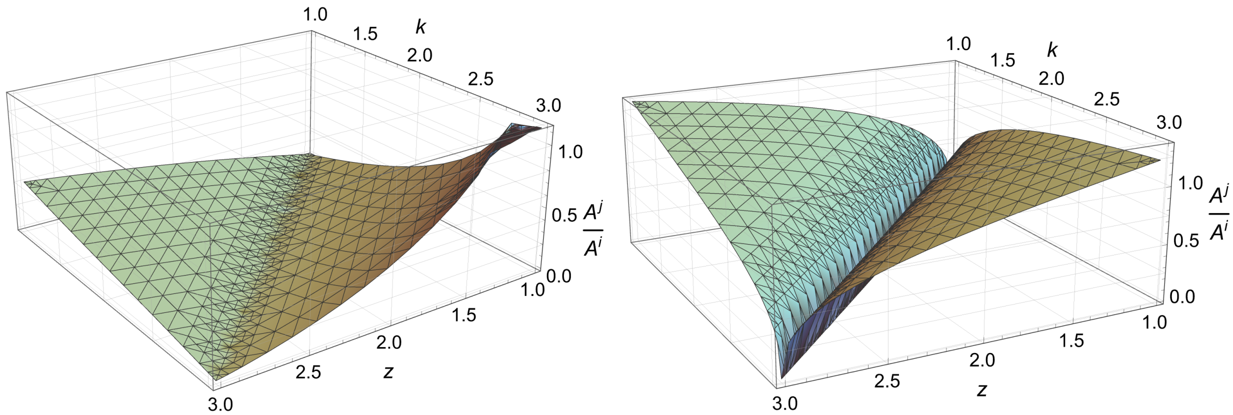


Figure C2: **Pooled investment rate with separate factor markets.** The above figure pools the average investment rate over 5,000 sample paths for the optimal $I_{k^i}{}^*, I_{z^i}{}^*, I_{k^j}{}^*, I_{z^j}{}^*$ over 10 years.

over the four types. The optimal investment rate is solved numerically and plots the average over 5,000 sample paths over 10 years. The value of φ is chosen such that the average investment rate is ~ 0.22, to match the data. Consistent with the results in Section 2.3, investment is increasing in volatility.

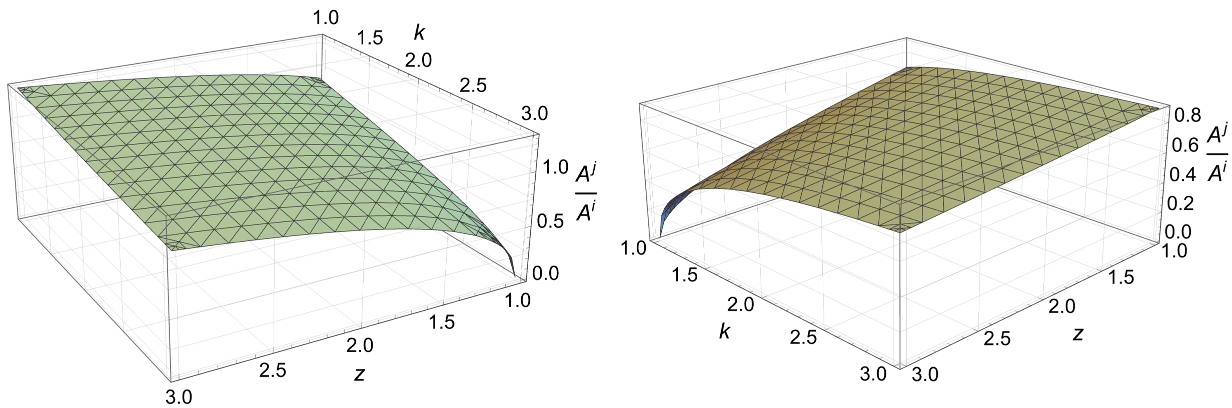
D Appendix: Supplementary Figures and Tables



(a) $\sigma^i = 0.12$ and $\sigma^j = 0.05$.

(b) $\sigma^i = 0.05$ and $\sigma^j = 0.12$.

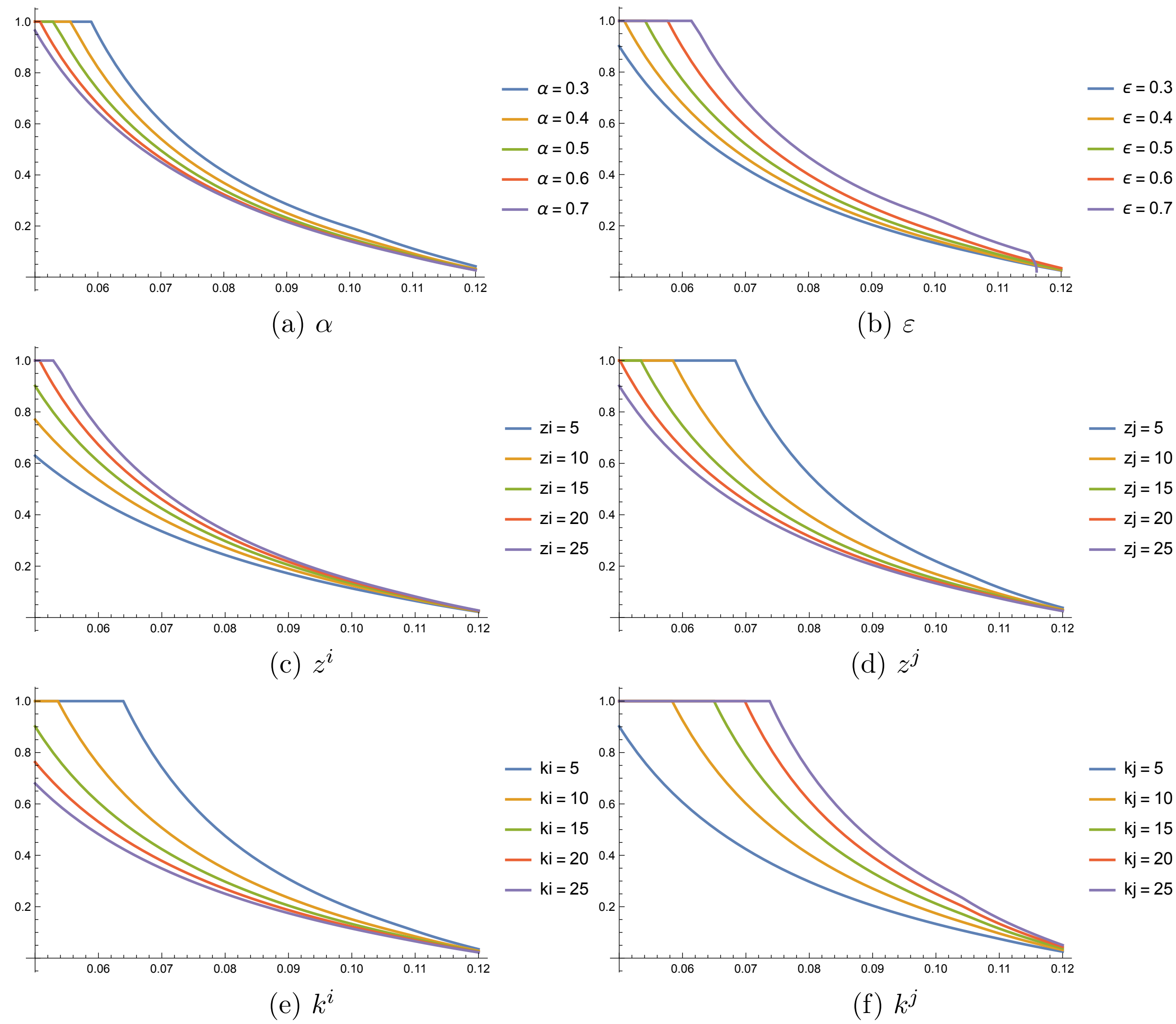
Figure IA.1: This figure repeats the exercise in Figure 3, but set volatility to be asymmetric between product markets i and j. Parameters are identical otherwise.



(a) $z^i = z^j = 5$ and $k^i = k^j = 15$

(b) $z^i = z^j = 15$ and $k^i = k^j = 5$

Figure IA.2: This figure repeats the exercise in Figure 3, but varies the firm's existing stock of general and specific capital. Parameters are identical otherwise.



(a) α (b) ε

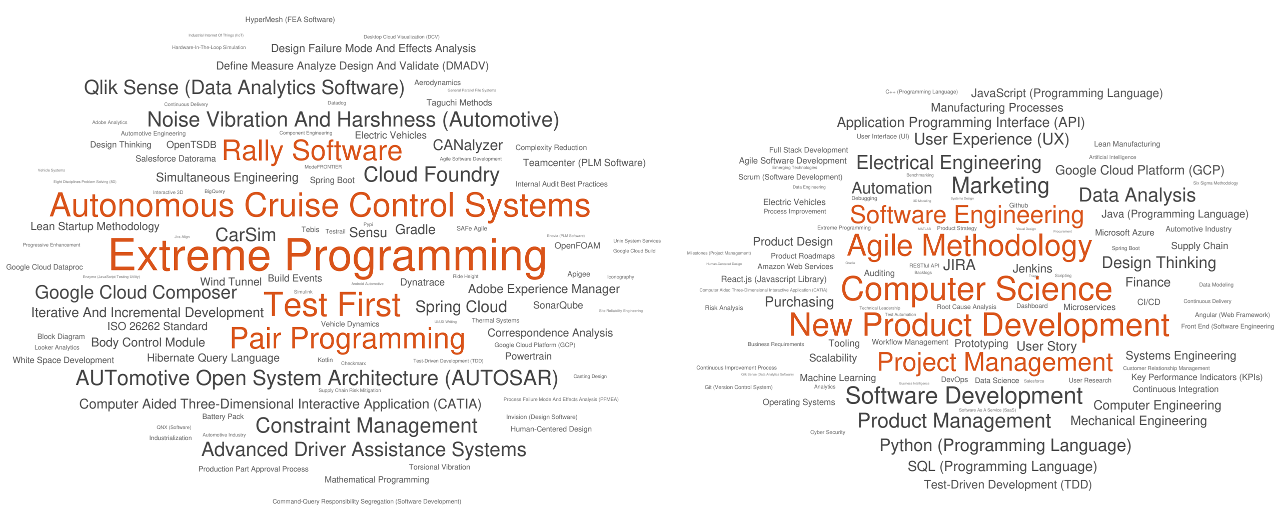
(c) z^i (d) z^j

(e) k^i (f) k^j

Figure IA.3: **Investment rates for different initial conditions.** The above figures show the investment rate stated in Proposition 4 for different initial conditions. Unless the parameter is varied as shown in the legend, the default parameter values are $\alpha = 0.4, \varepsilon = 0.6, \theta = 1, z^i = 15, z^j = 25, k^i = 15, k^j = 5$. In all figures, $\sigma^j = 0.06$ while σ^i varies along the x-axis. Moreover, $r = 0.035, \mu^i = 0.04, \mu^j = 0.02, \rho = 0, \kappa^i = \kappa^j = 0.45$ and $c = 2$. The y-axis is the investment rate over 25 projects.

(a) Specific Skills (b) General Skills

Figure IA.4: **Skills in sic−62: Security & Commodity Brokers.**



(a) Specific Skills (b) General Skills

Figure IA.5: **Ford Motors, 2023.** The word clouds show the specific and general skills extracted from Ford Motor's job postings in 2023.

	Metal Mining,10	Coal Mining,12	Oil & Gas Extraction,13	Nonmetallic Minerals, Except Fuels,14	General Building Contractors,15	Heavy Construction, Except Building,16	Special Trade Contractors,17	Food & Kindred Products,20	Tobacco Products,21	Textile Mill Products,22	Apparel & Other Textile Products,23	Lumber & Wood Products,24	Furniture & Fixtures,25	Paper & Allied Products,26	Printing & Publishing,27	Chemicals & Allied Products,28	Petroleum & Coal Products,29	Rubber & Miscellaneous Plastic Products,30	Leather & Leather Products,31	Stone, Clay & Glass Products,32	Primary Metal Industries,33	Fabricated Metal Products,34	Industrial Machinery & Computer Equipment,35	Electronic & Other Electric Equipment,36	Transportation Equipment,37	Measuring, Medical & Optical Equipment,38	Miscellaneous Manufacturing Industries,39	Railroad Transportation,40
Metal Mining,10	100	45	12	42	14	25	19	3	2	2	2	4	4	4	1	2	10	4	1	14	11	8	7	5	5	5	7	2
Coal Mining,12	45	100	13	50	21	39	28	3	3	2	2	5	5	4	1	2	8	5	1	13	8	8	7	6	6	6	7	2
Oil & Gas Extraction,13	12	13	100	16	11	20	12	2	2	1	1	4	5	4	0	3	33	4	1	6	7	10	13	6	8	6	9	2
Nonmetallic Minerals, Except Fuels,14	42	50	16	100	19	38	26	4	3	2	2	5	4	6	1	2	25	4	1	50	9	8	8	6	5	6	8	6
General Building Contractors,15	14	21	11	19	100	42	60	6	5	4	4	14	11	7	1	6	13	8	3	15	13	16	12	11	9	11	15	3
Heavy Construction, Except Building,16	25	39	20	38	42	100	54	4	3	2	2	7	7	6	1	3	17	6	2	19	14	14	11	11	8	9	13	14
Special Trade Contractors,17	19	28	12	26	60	54	100	6	4	4	4	13	14	8	1	5	31	8	3	17	15	18	15	14	11	13	16	9
Food & Kindred Products,20	3	3	2	4	6	4	6	100	18	4	4	7	7	11	1	7	5	8	4	10	12	11	7	6	3	7	14	2
Tobacco Products,21	2	3	2	3	5	3	4	18	100	2	5	6	6	8	1	7	2	7	6	4	6	6	5	4	3	7	8	1
Textile Mill Products,22	2	2	1	2	4	2	4	4	2	100	5	4	4	11	1	2	2	5	5	5	6	6	5	4	3	3	7	1
Apparel & Other Textile Products,23	2	2	1	2	4	2	4	4	5	5	100	4	5	7	1	2	2	52	62	4	4	7	5	4	3	4	8	1
Lumber & Wood Products,24	4	5	4	5	14	7	13	7	6	4	4	100	81	18	1	22	5	52	4	12	13	21	15	14	10	45	24	2
Furniture & Fixtures,25	4	5	5	4	11	7	14	7	6	4	5	81	100	14	1	20	5	47	5	13	15	26	27	27	14	51	24	2
Paper & Allied Products,26	4	4	4	6	7	6	8	11	8	11	7	18	14	100	3	7	7	14	7	13	15	18	11	9	6	11	22	2
Printing & Publishing,27	1	1	0	1	1	1	1	1	1	1	1	1	1	3	100	1	1	1	1	1	1	2	2	2	1	2	3	0
Chemicals & Allied Products,28	2	2	3	2	6	3	5	7	7	2	2	22	20	7	1	100	5	14	2	7	6	14	9	6	5	28	23	1
Petroleum & Coal Products,29	10	8	33	25	13	17	31	5	2	2	2	5	5	7	1	5	100	8	1	22	12	11	11	9	6	8	17	3
Rubber & Miscellaneous Plastic Products,30	4	5	4	4	8	6	8	8	7	5	52	52	47	14	1	14	8	100	38	10	12	17	12	10	9	28	33	1
Leather & Leather Products,31	1	1	1	1	3	2	3	4	6	5	62	4	5	7	1	2	1	38	100	3	3	5	5	3	2	3	7	0
Stone, Clay & Glass Products,32	14	13	6	50	15	19	17	10	4	5	4	12	13	13	1	7	22	10	3	100	24	32	19	17	15	13	19	4
Primary Metal Industries,33	11	8	7	9	13	14	15	12	6	6	4	13	15	15	1	6	12	12	3	24	100	42	25	25	17	20	24	2
Fabricated Metal Products,34	8	8	10	8	16	14	18	11	6	6	7	21	26	18	2	14	11	17	5	32	42	100	43	31	34	32	43	3
Industrial Machinery & Computer Equipment,35	7	7	13	8	12	11	15	7	5	5	5	15	27	11	2	9	11	12	5	19	25	43	100	42	24	29	33	3
Electronic & Other Electric Equipment,36	5	6	6	6	11	11	14	6	4	4	4	14	27	9	2	6	9	10	3	17	25	31	42	100	27	38	29	2
Transportation Equipment,37	5	6	8	5	9	8	11	3	3	3	3	10	14	6	1	5	6	9	2	15	17	34	24	27	100	32	16	2
Measuring, Medical & Optical Equipment,38	5	6	6	6	11	9	13	7	7	3	4	45	51	11	2	28	8	28	3	13	20	32	29	38	32	100	34	2
Miscellaneous Manufacturing Industries,39	7	7	9	8	15	13	16	14	8	7	8	24	24	22	3	23	17	33	7	19	24	43	33	29	16	34	100	2
Railroad Transportation,40	2	2	2	6	3	14	9	2	1	1	1	2	2	2	0	1	3	1	0	4	2	3	3	2	2	2	2	100

Figure IA.6: **Specialized skill relatedness across industries.** The x- and y-axis correspond to select 2-digit SIC codes. For a given pair of industries, the above plot shows the cosine similarity between each industry's vector of specialized skills. Data is from Lightcast IO, 2010 – 2023. An industry's vector of specialized skills is constructed as follows: I first aggregate job postings data and extract the required skills inferred from each posting. I apply the 'term frequency – inverse document frequency' (tf-idf) approach, weighting skills by how often one skill appears within postings posted by firms within one industry, then down-weighted by how often it appears in all other industries.

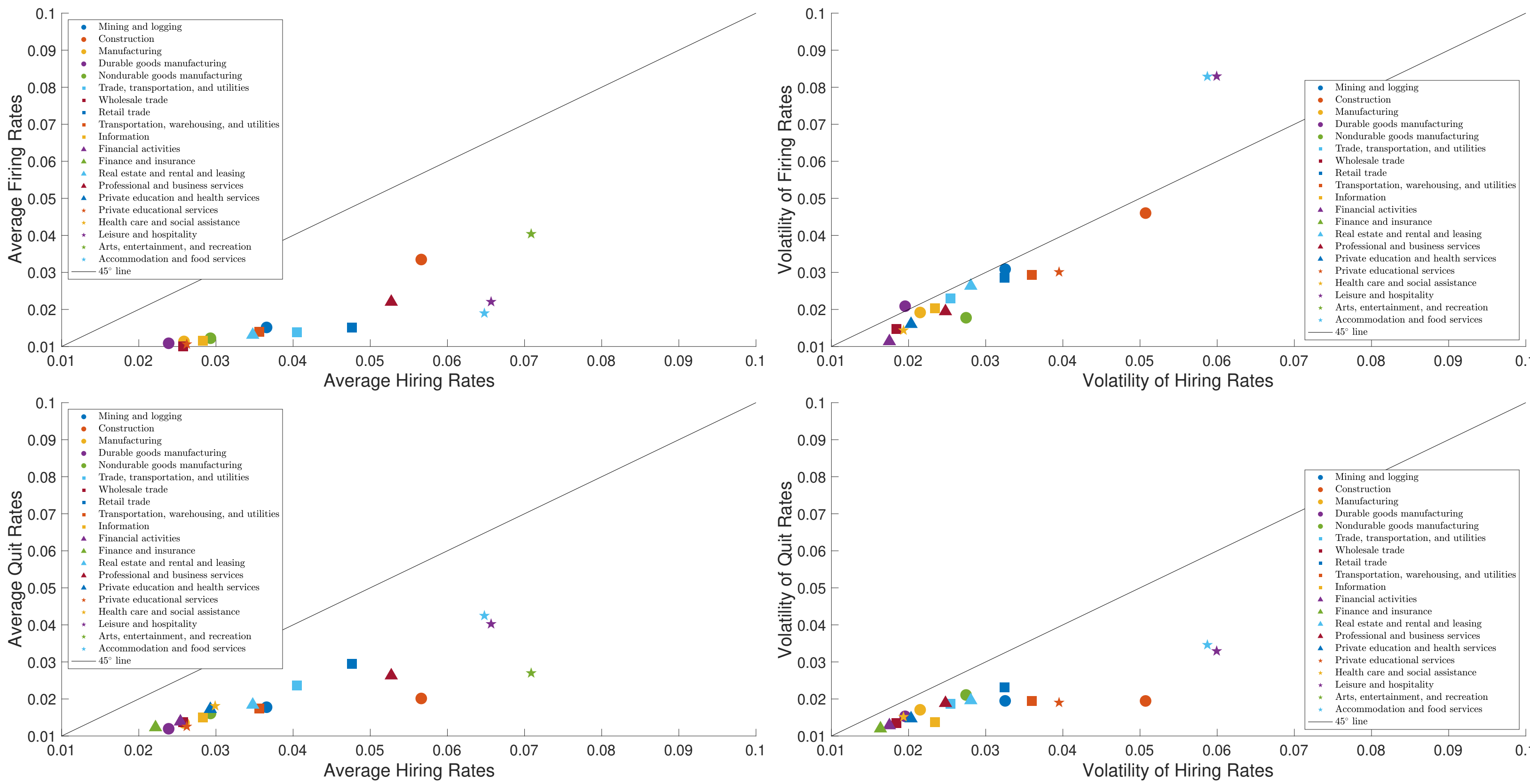


Figure IA.7: **Hire, Fire, Quit Rates by Major Industry.** Data is from the Bureau of Labor Statistics Job Openings & Labor Turnover Survey (JOLTS), Dec 2000 - June 2024. For each major industry, I plot the annualized mean and standard deviation hiring rates against quitting and firing rates.

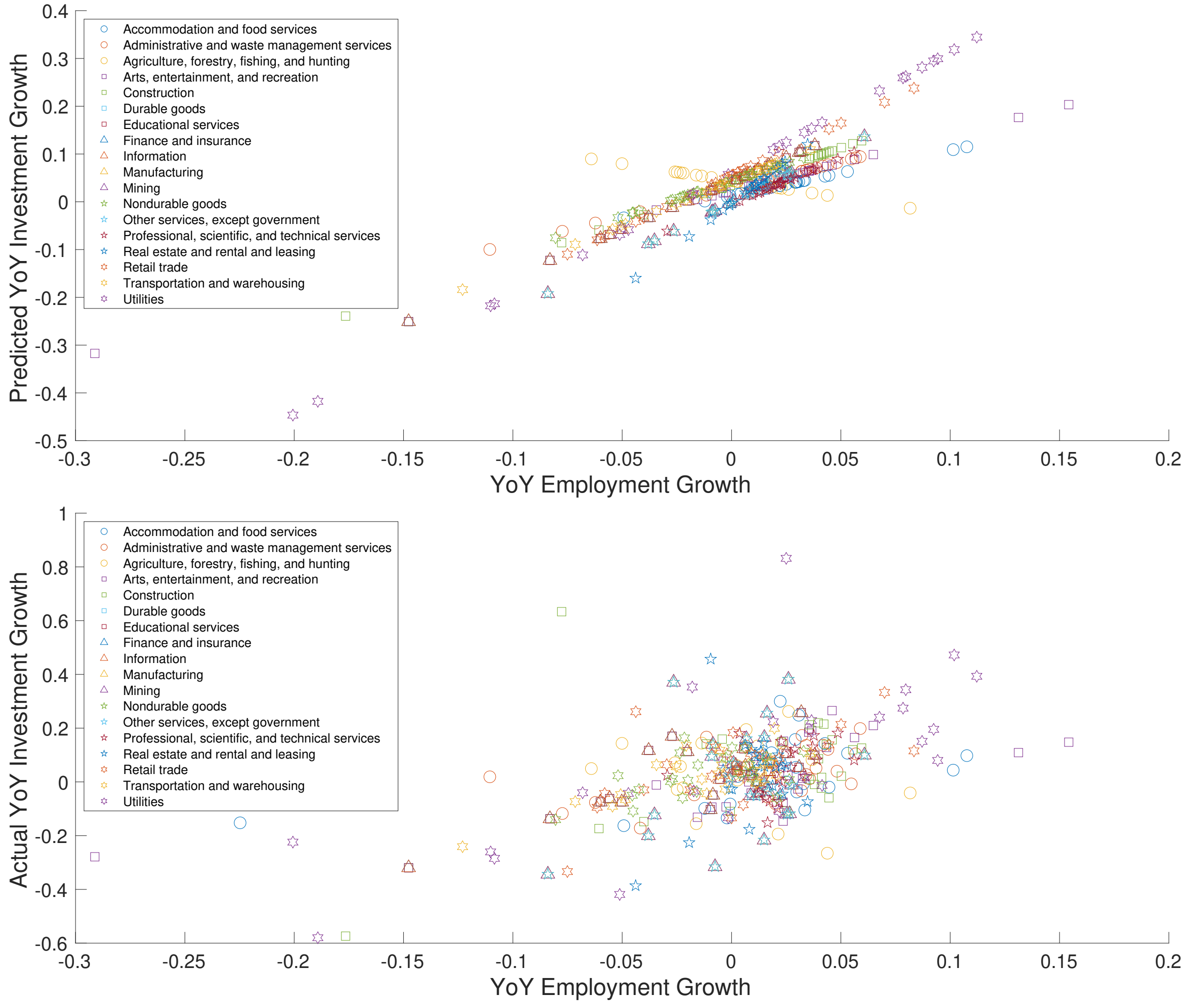


Figure IA.8: **Investment and Employment Growth by Major Industry.** The above figures are scatter plots of time t growth in full-time employment by industry against time t growth in investment in fixed assets: private equipment by industry. The upper figure uses predicted values from a simple regression for each industry and lower figure is the raw data. Data is annual, 1998-2024.

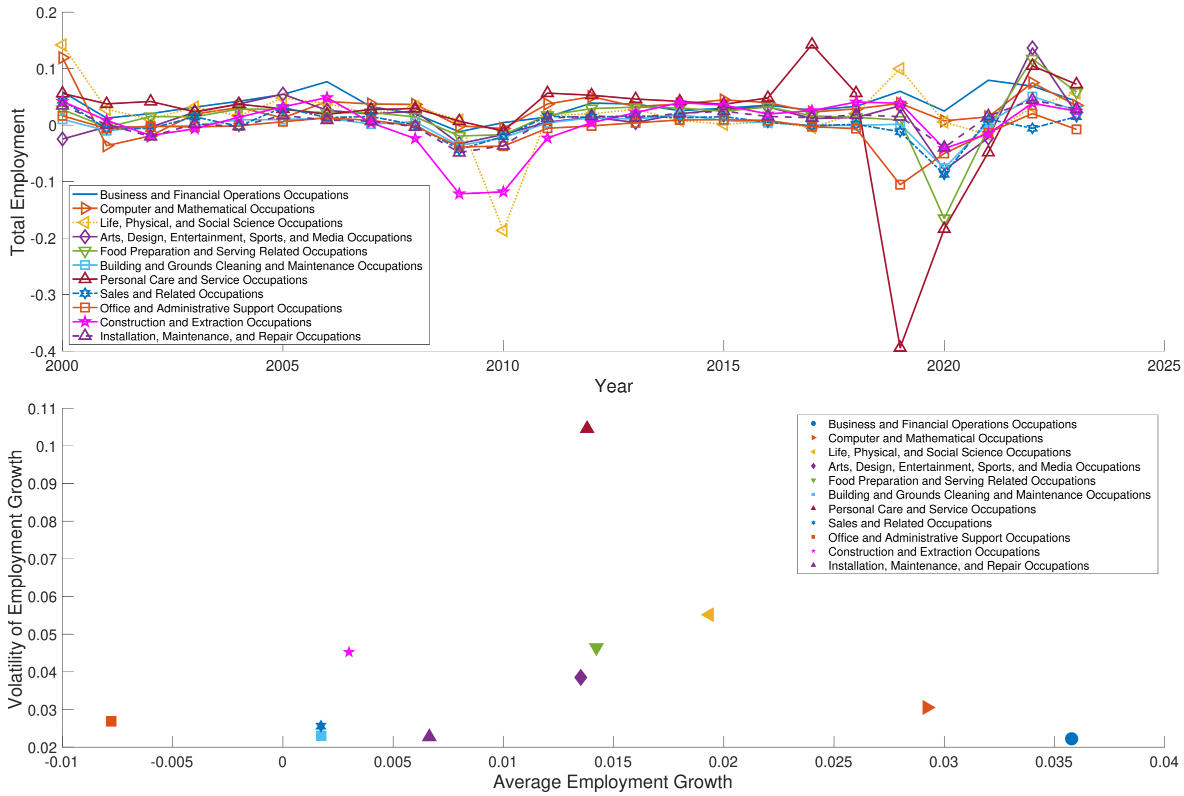


Figure IA.9: **Employment by Major Occupation Category.** The above figures show statistics on employment for select occupation categories using standard occupation codes (SOC) from the BLS. Data is 1999-2003. The upper plot shows the time-series for total employment while the lower plot shows the average employment growth rate over this time period, against the volatility of employment growth.

2-digit SIC

	13	20	25	26	28	29	30	32	33	34	35	36	37	38	39	48	50	59	73	87
13	25	2	2	1	1	5	1	2	2	2	2	1	2	2	1	1	2	0	0	1
20	2	10	1	3	4	3	2	2	2	2	1	0	1	1	1	0	3	1	0	2
25	2	1	22	3	0	1	2	2	1	5	2	2	4	2	4	2	2	2	1	1
26	1	3	3	16	2	2	3	4	2	3	1	1	1	2	1	0	3	1	1	1
28	1	4	0	2	14	6	1	2	3	1	1	1	1	2	1	0	6	0	0	5
29	5	3	1	2	6	16	2	2	11	1	1	1	2	2	1	1	7	0	0	2
30	1	2	2	4	1	2	19	2	2	3	1	1	2	2	2	1	3	1	1	1
32	2	2	2	4	2	2	2	14	2	2	1	1	2	2	1	1	3	1	0	1
33	2	2	1	2	3	11	2	2	15	2	1	1	2	1	1	1	4	1	0	2
34	2	2	5	3	1	1	3	2	2	11	2	2	5	2	2	1	2	1	1	2
35	2	1	2	1	1	1	1	1	1	3	6	5	3	3	2	4	2	5	4	2
36	1	0	2	1	1	1	1	1	1	2	5	9	2	3	1	4	2	3	2	2
37	2	1	4	1	1	2	2	2	2	5	3	2	12	2	3	2	2	1	1	1
38	2	1	2	2	2	2	2	2	1	2	3	3	2	7	2	2	3	1	1	2
39	1	1	4	1	1	1	2	1	1	2	2	1	3	2	24	1	2	2	2	1
48	1	0	2	1	0	1	1	1	1	1	5	4	2	2	2	19	1	13	8	5
50	2	3	2	3	6	7	3	3	4	2	2	2	2	3	2	1	12	3	1	2
59	1	1	2	1	0	0	1	1	1	1	5	3	1	1	2	13	3	20	12	8
73	0	0	1	1	0	0	1	0	0	1	4	2	1	1	2	8	1	12	10	5
87	1	2	1	1	5	2	1	1	2	2	2	2	1	2	1	5	2	8	5	9

Figure IA.10: **Relatedness across industries.** The x- and y-axis correspond to select 2-digit SIC codes. For a given row and column, this figure shows the average cosine similarity between patents filed by single-segment firms from each respective industry that were filed within ± 5 years of each other. Values are expressed in percent units.
2-digit SIC code classifications. **13**: Oil & Gas Extraction, **20**: Food & Kindred Products, **25**: Furniture & Fixtures, **26**: Paper & Allied Products, **28**: Chemical & Allied Products, **29**: Petroleum & Coal Products, **30**: Rubber & Misc Plastics Products, **32**: Stone, Clay, & Glass Products, **33**: Primary Metal Industries, **34**: Fabricated Metal Products, **35**: Industrial Machinery & Equipment, **36**: Electronic & Other Electric Equipment, **37**: Transportation Equipment, **38**: Instruments & Related Products, **39**: Misc Manufacturing Industries, **48**: Communications, **50**: Wholesale Trade - Durable Goods, **59**: Misc Retail, **73**: Business Services, **87**: Engineering & Management Services.

Table IA.1: **Volatility of volatility.** Data is from 1976-2022. This table shows summary statistics on the within-firm standard deviation of firm-level uncertainty measures in the merged CRSP-Compustat data. Vol 5 is the annualized standard deviation of excess stock returns over 5 year rolling windows at the end of June. Vol 1 uses 1 year rolling windows. The left panel is the total sample for which volatility measures can be computed while the right is the final sample that fit various criteria (such as non-missing assets, sales, etc).

	Raw		Final	
Percentile	Vol 5	Vol 1	Vol 5	Vol 1
5th	0.01	0.03	0.01	0.04
25th	0.04	0.09	0.04	0.08
50th	0.07	0.15	0.07	0.12
75th	0.13	0.26	0.12	0.19
95th	0.29	0.54	0.23	0.37
Average	0.1	0.21	0.09	0.16
std	0.12	0.22	0.09	0.14
Fim-year obs	188,615	282,094	65,369	65,369

Table IA.2: **Total Skill Frequency.** Data is 2010-2023 for approximately 270 million job vacancy postings in Lightcast. One posting can contain multiple skills, but does not contain duplicates. The column "Frequency" corresponds to the number of unique job postings that asked for the skill. N corresponds to the number of 2-digit SIC codes in which this skilled appeared over the data sample.

Rank	Skill	Frequency	N
1	Project Management	19,719,954	82
2	Marketing	17,774,055	82
3	Nursing	15,186,615	82
4	Auditing	13,665,367	82
5	Accounting	13,183,871	82
6	Merchandising	12,848,564	82
7	Finance	11,831,944	82
8	Selling Techniques	10,868,076	82
9	Computer Science	10,548,703	82
10	Warehousing	8,385,302	82
11	SQL (Programming Language)	7,747,261	82
12	Workflow Management	7,530,709	82
13	Process Improvement	7,258,119	82
14	Data Analysis	7,096,024	82
15	Agile Methodology	6,846,713	82
16	Customer Relationship Management	6,656,442	82
17	Automation	6,274,417	82
18	Restaurant Operation	6,159,469	82
19	Billing	6,071,707	82
20	Sales Prospecting	5,952,845	82
21	Invoicing	5,710,526	82
22	Purchasing	5,507,525	82
23	New Product Development	5,462,659	82
24	Housekeeping	5,455,420	82
25	Java (Programming Language)	5,286,754	82
26	Business Development	5,268,529	82
27	Product Knowledge	5,185,472	82
28	Data Entry	5,147,143	82
29	Inventory Management	5,094,170	82
30	Financial Statements	4,788,888	82
31	General Mathematics	4,751,771	82
32	Business Process	4,734,427	82
33	Software Development	4,679,602	82
34	Forklift Truck	4,545,117	82
35	Medical Records	4,525,049	82
36	Construction	4,476,635	82
37	Effective Communication	4,429,798	82
38	Financial Services	4,408,307	82
39	Risk Management	4,378,151	82
40	Business Requirements	4,362,263	82
41	Python (Programming Language)	4,229,931	82
42	JavaScript (Programming Language)	4,185,204	82
43	Operating Systems	4,120,404	82
44	Nursing Care	4,081,302	82
45	SAP Applications	3,976,192	82
46	Cash Handling	3,932,406	82
47	Cash Register	3,916,701	82
48	Software Engineering	3,899,709	82
49	Procurement	3,856,464	82
50	Information Systems	3,724,863	82

Year	# Firms	Average	Std	p5	p25	p50	p75	p95
2010	3,895	557	2,587	1	5	38	224	2,073
2011	4,344	659	3,255	1	6	40	267	2,540
2012	4,230	743	3,239	1	7	60	340	3,037
2013	4,845	804	4,011	1	7	58	319	2,924
2014	4,649	953	4,914	1	8	68	355	3,450
2015	4,793	1,139	6,661	1	7	57	368	4,422
2016	4,632	1,291	7,859	1	7	60	399	4,812
2017	4,778	1,214	8,983	1	7	59	393	3,929
2018	5,625	1,261	9,281	1	8	65	429	4,144
2019	5,818	1,140	7,076	1	8	66	425	3,951
2020	5,539	1,045	6,771	1	8	62	365	3,387
2021	5,561	1,320	7,036	1	11	77	495	4,960
2022	5,579	1,435	7,237	1	12	83	543	5,420
2023	5,296	1,052	4,889	1	10	67	413	4,087

Table IA.3: Number of postings each year for matched Compustat firms in Lightcast. This table shows summary statistics on the number of unique job postings collected in the Lightcast database for firms matched to Compustat.

Year	Average	Std	p5	p25	p50	p75	p95
2010	2.44	1.81	0.48	1.12	2.06	3.24	5.74
2011	2.21	1.59	0.45	1.09	1.91	2.92	4.87
2012	2.14	1.51	0.40	1.10	1.85	2.78	4.99
2013	1.99	1.27	0.45	1.03	1.77	2.65	4.34
2014	2.10	1.37	0.46	1.07	1.82	2.88	4.44
2015	2.08	1.43	0.46	1.05	1.77	2.85	4.57
2016	1.83	1.28	0.34	0.89	1.60	2.48	4.03
2017	2.00	1.30	0.42	1.01	1.76	2.70	4.43
2018	1.96	1.23	0.43	1.06	1.72	2.63	4.24
2019	2.01	1.28	0.42	1.07	1.79	2.73	4.29
2020	1.77	1.15	0.36	0.91	1.58	2.34	3.91
2021	1.90	1.22	0.37	0.99	1.68	2.60	4.07
2022	1.90	1.23	0.36	0.98	1.65	2.62	4.12
2023	1.80	1.17	0.36	0.97	1.54	2.42	3.97
Full Sample	2.00	1.36	0.41	1.02	1.73	2.69	4.45

Table IA.4: **Summary Statistics**: $\text{General}_{i,t}$

Table IA.5: This table is the panel regression:

$$\text{R\&D Investment rate}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \gamma_{j,t} + \varepsilon_{i,t+1}$$

The dependent variable is the R&D investment rate at $t+1$, calculated as R&D expenditures divided by the average of current and lagged stock of *knowledge capital*. This regression repeats the analysis from Table 3 but drops firm fixed-effects due to the inclusion of lagged R&D investment at t as a control.

Dependent Variable: R&D Investment $\text{rate}_{i,t+1}$

	1.	2.	3.	4.	5.	6.	7.
Vol_{5y}	2.30***	2.10***	2.33***				
	(10.19)	(6.92)	(10.74)				
Abnormal Vol_{5y}				2.54***			
				(10.47)			
Vol_{5y} (de-levered)					3.11***		
					(9.11)		
Vol_{5y} (monthly)						2.07***	
						(5.77)	
Cashflow Vol_{5y}							2.30***
							(4.27)
Firm controls				Yes			
Fixed effects	Year	ff-49 × Year	sic-2 × Year	sic-2 × Year			
Standard errors	Firm, Year	ff-49, Year	sic-2, Year	sic-2, Year			
Adjusted R^2	0.81	0.82	0.81	0.81	0.81	0.81	0.81
Observations	40,384	40,384	40,384	40,384	40,384	40,384	41,041

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table IA.6: This table is the panel regression:

$$\Delta\text{Flow to Intangibles}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

for two dependent variables. The left panel is the change in R&D expenditures and the right panel is the change in intangible investment, as defined in Peters and Taylor 2017. The dependent variables are computed in the following way: $\Delta y_{i,t+1} = (y_{i,t+1} - y_{i,t})/(.5y_{i,t+1} + .5y_{i,t})$, which bounds these growth rates between $[-2, 2]$ for positive values of y, following Alfaro et al. 2024. Moreover, I require $\Delta\text{Intangible Investment}_{i,t+1}$ to have non-missing R&D data in the right panel. The independent variables include a vector of firm controls $\mathbf{x}_{i,t}$, firm i and industry $j \times$ year t fixed effects, and the main variable of interest: firm volatility at t. Vol_{5y} is the annualized volatility on the firm's excess stock returns using daily data from $t-5$ to t as of June for that fiscal year, then lagged by one year to avoid possible concerns with reverse causality. Within a calendar year, stock returns are required to have a minimum of 230 observations. Firm controls include: Tobin's Q, the Whited-Wu index, log sales, payout, tangibility (Leary and Roberts 2014), return on assets, book leverage, log assets, intangible investment at t (Peters and Taylor 2017), investment rate I/K at t, r&d/assets, capex/assets, firm and primary industry sales growth. Moreover, following Alfaro et al. 2024, I include the compounded excess stock returns over the 5 year period as a first-moment control when incorporating return based measures of volatility. Data variables are winsorized at the 1st and 99th percentiles. Coefficients are multiplied ×100.

Dependent Variable: $\Delta\text{Flow to Intangibles}_{i,t+1}$

	$\Delta\text{R\&D Expenditure}_{i,t+1}$			$\Delta\text{Intangible Investment}_{i,t+1}$		
	1.	2.	3.	4.	5.	6.
Vol_{5y}	5.97***	6.80**	7.10**	5.10***	5.68**	6.13**
	(3.21)	(2.27)	(2.58)	(3.78)	(2.46)	(2.46)
Firm controls		Yes			Yes	
Fixed effects	Firm, Year	Firm, sic-2 × Year	Firm, ff-49 × Year	Firm, Year	Firm, sic-2 × Year	Firm, ff-49 × Year
Standard errors	Firm, Year	sic-2, Year	ff-49, Year	Year	sic-2, Year	ff-49, Year
Adjusted R^2	0.11	0.12	0.12	0.24	0.25	0.25
N	44444	44119	44302	40742	40372	40561

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table IA.7: This table is the panel regression:

$$\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

I repeat the analysis from Table 4 but uses different measures of firm-level risk. Cashflow Vol_{5y} is the rolling 5-year sales volatility using quarterly data, scaled by lagged assets. Cashflow CV_{5y} scales by average sales over time, rather than lagged assets. Sales Forecast Vol_{3y} is the the volatility of I/B/E/S analyst sales forecasts made at t for sales at $t+3$, scaled by lagged assets. Sales Forecast CV_{3y} is the coefficient of variation. I require at least 2 analysts for a given firm.

Dependent Variable: $\text{General}_{i,t+1}$

	(1)	(2)	(3)	(4)
Cashflow Vol_{5y}	-0.40***			
	(-2.92)			
Cashflow CV_{5y}		-0.15**		
		(-2.47)		
Analyst Sales Forecast Vol_{3y}			-0.16**	
			(-2.43)	
Analyst Sales Forecast CV_{3y}				-0.07*
				(-1.91)
Fixed effects	Firm, Industry × Year			
Standard errors	Industry, Year			
Adjusted R^2	0.70	0.70	0.69	0.69
N	16,711	16,711	13,730	13,730

t statistics in parentheses
* $p < 0.10$,
** $p < 0.05$
*** $p < 0.01$

Table IA.8: This table repeats the analysis in Table 4 using different fixed effects specifications. The panel regression is:

$$\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta\text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

where the dependent variable $\text{General}_{i,t+1}$ measures how similar the desired skills in firm i's time $t+1$ job vacancy postings are to the specialized skill vectors in all industries outside of firm i's own. The independent variables are the same as before. In the first three columns, I use firm and Industry × Year fixed effects, where industry is defined according to Fama-French 49 industry classifications. In the next three columns, I use Firm and Year fixed effects. Standard errors are double clustered by Year and either Firm or Industry.

Dependent Variable: $\text{General}_{i,t+1}$

	1.	2.	3.	4.	5.	6.
Vol_{5y}	-0.26**			-0.27*		
	(-2.69)			(-2.06)		
Abnormal Vol_{5y}		-0.35***			-0.34**	
		(-3.25)			(-2.35)	
Vol_{5y} (de-levered)			-0.32**			-0.34*
			(-2.50)			(-1.95)
Firm controls		Yes			Yes	
Fixed effects		Firm, ff-49 × Year			Firm, Year	
Standard errors		ff-49, Year			Firm, Year	
Adjusted R^2	0.70	0.70	0.70	0.69	0.69	0.68
Observations	16,706	16,706	16,706	16,711	16,711	16,711

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table IA.9: This table repeats the analysis in Table 4 keeping only firm-level risk and fixed effects as control variables. The panel regression is:

$$\text{General}_{i,t+k} = \beta_1 \text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

for $k \in \{1, 2\}$ years. The dependent variable $\text{General}_{i,t+k}$ measures how similar the desired skills in firm i's time $t+k$ job vacancy postings are to the specialized skill vectors in all industries outside of firm i's own. It is the average cosine similarity over firm i and each outside industry's vector of skills using the term-frequency-inverse-document-frequency (tf-idf) approach. Further details are in **??**. The independent variables include a vector of firm controls $\mathbf{x}_{i,t}$, firm i and industry j × year t fixed effects, and the main variable of interest: firm volatility at t. Vol_{5y} is the annualized volatility on the firm's excess stock returns using daily data from $t-5$ to t as of June for that fiscal year, then lagged by one year. Following Kermani and Ma 2023, Abnormal Vol_{5y} is the volatility of residuals from a regression of firm i excess stock returns on the Fama-French five-factor model, also lagged by one year. Vol_{5y} (de-levered) is Vol_{5y} divided by 1 plus the firm's gross debt-equity ratio. Within a calendar year, stock returns are required to have a minimum of 230 observations. Firm controls include: Tobin's Q, the Whited-Wu index, log sales, payout, tangibility (Leary and Roberts 2014), return on assets, book leverage, log assets, intangible investment at t (Peters and Taylor 2017), investment rate I/K at t, r&d/assets, capex/assets, firm and primary industry sales growth. Moreover, following Alfaro et al. 2024, I include the compounded excess stock returns over the 5 year period as a first-moment control when incorporating return based measures of volatility. Data variables are winsorized at the 1st and 99th percentiles. Coefficients are multiplied ×100. Data is annual, 2010 − 2023.

Dependent Variable: $\text{General}_{i,t+k}$

	$\text{General}_{i,t+1}$			$\text{General}_{i,t+2}$		
	1.	2.	3.	4.	5.	6.
Vol_{5y}	-0.37***			-0.37***		
	(-3.23)			(-3.37)		
Abnormal Vol_{5y}		-0.48***			-0.47***	
		(-3.61)			(-4.10)	
Vol_{5y} (de-levered)			-0.13			-0.07
			(-1.06)			(-0.77)
Firm controls	Yes			Yes		
Fixed effects	Firm, sic-2 × Year			Firm, sic-2 × Year		
Standard errors	sic-2, Year			sic-2, Year		
Adjusted R^2	0.69	0.69	0.69	0.70	0.70	0.70
Observations	17,614	17,614	17,614	14,993	14,993	14,993

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table IA.10: This table repeats the left panel of Table 4 for alternative constructions of $\text{General}_{i,t+1}$. Columns 1&4 calculates the measure including all job postings without filtering out positions in Finance, Human Resources, and Law, Compliance, and Public Safety. Moreover, the set of "outside" industries $\mathscr{I}$ excludes all industries the firm reports in Compustat segments (instead of just the firm's primary industry). Columns 2&5 only excludes the firm's primary industry as assigned by Lightcast. Finally, columns 3&6 applies the filters and uses the firm's full set of reported industries in Compustat segments. The accounting variables shown are controls with large economic magnitudes in the regressions. For reference, the standard deviation of the General measure is between 1.2-1.4.

Dependent Variable: $\text{General}_{i,t+1}$

	1.	2.	3.	4.	5.	6.
Vol_{5y}	-0.21**	-0.23**	-0.25**			
	(-2.21)	(-2.33)	(-2.34)			
Vol_{5y} (de-levered)				-0.26*	-0.28**	-0.32**
				(-2.04)	(-2.20)	(-2.22)
Tobin's q	0.07**	0.07**	0.08***	0.06**	0.06**	0.07***
	(2.81)	(2.85)	(3.27)	(2.70)	(2.74)	(3.13)
Log Sales	0.15***	0.16***	0.14**	0.15***	0.15***	0.14**
	(4.07)	(3.86)	(2.77)	(3.90)	(3.70)	(2.64)
Log Assets	0.13***	0.14***	0.14***	0.13***	0.14***	0.14***
	(4.85)	(5.03)	(5.38)	(5.01)	(5.23)	(5.47)
Leverage	-0.13	-0.14	-0.10	-0.26**	-0.28**	-0.25**
	(-1.56)	(-1.66)	(-1.53)	(-2.58)	(-2.92)	(-2.47)
Firm controls	Yes					
Fixed effects	Firm, sic-2 × Year					
Standard errors	sic-2, Year					
Adjusted R^2	0.68	0.68	0.70	0.68	0.68	0.670
Observations	16,943	16,943	16,711	16,943	16,943	16,711

t statistics in parentheses

* $p < 0.10$

** $p < 0.05$

*** $p < 0.01$

Table IA.11: This table is the panel regression:

$$\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \beta_2 \mathbb{1}_{i,t,\text{Avg Dist.}>0} + \beta_3 \text{Volatility}_{i,t} \times \mathbb{1}_{i,t,\text{Avg Dist.}>0} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

I repeat the analysis from Table 4 but add an interaction term for the indicator variable that takes the value of 1 if the average distance is greater than 0 across all coordinate pairs (all job positions are based in the same city).

Dependent Variable: $\text{General}_{i,t+1}$

	1.	2.	3.	4.	5.	6.
$\mathbb{1}_{i,t,\text{Avg Dist.}>0}$	0.18**	0.39***	0.18**	0.41***	0.18**	0.35***
	(2.73)	(3.66)	(2.70)	(4.11)	(2.71)	(3.55)
Vol_{5y}	-0.23**	0.13				
	(-2.18)	(0.73)				
$\mathbb{1}_{i,t,\text{Avg Dist.}>0} \times \text{Vol}_{5y}$		-0.38**				
		(-2.28)				
Abnormal Vol_{5y}			-0.31**	0.14		
			(-2.60)	(0.73)		
$\mathbb{1}_{i,t,\text{Avg Dist.}>0} \times$ Abnormal Vol_{5y}				-0.47**		
				(-2.69)		
Vol_{5y} (de-levered)					-0.29*	0.08
					(-2.03)	(0.36)
$\mathbb{1}_{i,t,\text{Avg Dist.}>0} \times \text{Vol}_{5y}$ (de-levered)						-0.40*
						(-1.96)
Firm controls	Yes					
Fixed effects	Firm, sic-2 × Year					
Standard errors	sic-2, Year					
Adjusted R^2	0.70	0.70	0.70	0.70	0.70	0.70
Observations	16,711	16,711	16,711	16,711	16,711	16,711

t statistics in parentheses
* $p < 0.10$
** $p < 0.05$
*** $p < 0.01$

Table IA.12: This table is the panel regression:

$$\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta\text{Volatility}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

from Table 4 but splits the sample based on whether the average distance for firm i's job positions is above or below the mean across all firms at t.

Dependent Variable: $\text{General}_{i,t+1}$

	Above Mean Distance			Below Mean Distance		
	1.	2.	3.	4.	5.	6.
Vol_{5y}	-0.38**			-0.13		
	(-2.33)			(-0.62)		
Abnormal Vol_{5y}		-0.49**			-0.16	
		(-2.66)			(-0.93)	
Vol_{5y} (de-levered)			-0.36			-0.28
			(-1.54)			(-1.03)
Firm controls	Yes			Yes		
Fixed effects	Firm, sic-2 × Year			Firm, sic-2 × Year		
Standard errors	sic-2, Year			sic-2, Year		
Adjusted R^2	0.71	0.71	0.71	0.67	0.67	0.67
Observations	10,002	10,002	10,002	6,709	6,709	6,709

t statistics in parentheses
* $p < 0.10$
** $p < 0.05$
*** $p < 0.01$

Table IA.13: This table is the panel regression:

$$\text{General}_{i,t+1} = \mathbf{x}_{i,t}\boldsymbol{\xi} + \beta_1 \text{Volatility}_{i,t} + \beta_2 \text{Proxy}_{i,t} + \beta_3 \text{Volatility}_{i,t} \times \text{Proxy}_{i,t} + \delta_i + \gamma_{j,t} + \varepsilon_{i,t+1}$$

I repeat the analysis from Table 4 but use different proxy variables as validation checks. Firm Scope is a measure of scope from Hoberg and Phillips 2024, which measures the number of distinct product markets in which the firm operates. Avg Distance (filtered) is average distance as before, but filtering out job positions with middle and back office roles as well as jobs in sales.

Dependent Variable: $\text{General}_{i,t+1}$

	1.	2.	3.	4.
Vol_{5y}	-0.55	-0.49	-0.46**	0.09
	(-0.99)	(-1.11)	(-2.51)	(0.55)
Log Sales	0.21**			
	(2.75)			
Log Sales × Vol_{5y}	0.04			
	(0.44)			
Log Assets		0.21***		
		(3.67)		
Log Assets × Vol_{5y}		0.03		
		(0.43)		
Firm Scope			0.00	
			(0.34)	
Firm Scope × Vol_{5y}			0.01	
			(0.80)	
Avg Distance (filtered)				0.08***
				(4.71)
Avg Distance (filtered) × Vol_{5y}				-0.07**
				(-2.52)
Firm controls	Yes			
Fixed effects	Firm, sic-2 × Year			
Standard errors	sic-2, Year			
Adjusted R^2	0.70	0.70	0.70	0.70
Observations	16,711	16,711	14,583	16,503

t statistics in parentheses
* $p < 0.10$
** $p < 0.05$
*** $p < 0.01$

Table IA.14: This table is the panel regression:

$$\text{Investment Growth in Fixed Assets}_{j,t} = \beta \text{Employment Growth}_{j,t} + \delta_j + \gamma_t + \varepsilon_{j,t}$$

The dependent variable is the investment growth in fixed assets at t for each major industry category j from the Bureau of Economic Analysis. The columns in the regression table add up the data series used: 1, Investment in Private Equipment by Industry. 2, Investment in Private Structures by Industry. 3, Investment in Private Intellectual Property Products by Industry. The main independent variable is the YoY growth in employment, from the series: Full-Time Equivalent Employees by Industry. Data is 1998-2022.

Dependent Variable: Investment Growth in Fixed Assets$_{i,t}$

	1, Equipment	1+ 2, Structures	1+2+3, IP
Employment Growth$_{j,t}$	1.27**	1.06**	1.28***
	(2.57)	(2.46)	(3.05)
Fixed effects		Firm, Industry × Year	
Standard errors		Industry, Year	
Adjusted R^2	0.34	0.28	0.31
N	408	408	408

t statistics in parentheses

* $p < 0.10$,

** $p < 0.05$

*** $p < 0.01$

Table IA.15: Average $General_p$ by Industry. Calculated using Equation B2. This table shows the average $General_p$ score for patents filed in a given industry. Construction is detailed in Appendix Section B.3. Scores are out of 100. The final column "Avt Pct" corresponds to the average fraction of patents outside of a patent's classified industry that exceed 5% cosine similarity.

sic-2	Industry Description	Count	General	Avg Pct.
59	Miscellaneous Retail	19,103	8.30	0.09
48	Communications	69,761	6.94	0.07
78	Motion Pictures	5,959	6.45	0.07
51	Wholesale Trade - Nondurable Goods	4,026	6.09	0.06
35	Industrial Machinery and Computer Equipment	395,845	5.97	0.06
60	Depository Institutions	11,005	5.61	0.06
67	Holding and Other Investment Offices	26,857	5.60	0.06
50	Wholesale Trade - Durable Goods	11,225	5.50	0.06
63	Insurance Carriers	5,189	5.34	0.06
61	Nondepository Credit Institutions	20,834	5.32	0.06
12	Coal Mining	1,558	5.18	0.06
36	Electronic and Other Electric Equipment	863,980	5.09	0.05
27	Printing and Publishing	4,827	4.99	0.05
87	Engineering, Accounting, Research, and Mgmt Services	14,434	4.99	0.05
38	Measuring, Medical and Optical Equipment	314,402	4.96	0.05
42	Motor Freight Transportation	1,508	4.81	0.05
79	Amusement and Recreation Services	2,881	4.77	0.05
29	Petroleum and Coal Products	44,937	4.51	0.05
73	Business Services	277,456	4.34	0.05
49	Electric, Gas and Sanitary Services	4,116	4.25	0.05
65	Real Estate	2,946	4.13	0.04
34	Fabricated Metal Products	19,863	3.95	0.04
53	General Merchandise Stores	1,167	3.82	0.04
33	Primary Metal Industries	16,979	3.74	0.04
16	Heavy Construction, Except Building	2,946	3.54	0.04
32	Stone, Clay and Glass Products	12,405	3.51	0.04
62	Security & Commodity Brokers, Dealers, Exch. & Services	2,648	3.41	0.04
39	Miscellaneous Manufacturing Industries	14,680	3.40	0.04
10	Metal Mining	1,430	3.39	0.04
24	Lumber and Wood Products	2,367	3.32	0.04
25	Furniture and Fixtures	8,397	3.29	0.04
30	Rubber and Miscellaneous Plastic Products	21,035	3.24	0.04
37	Transportation Equipment	235,804	3.20	0.03
26	Paper and Allied Products	27,818	3.00	0.03
22	Textile Mill Products	2,548	2.94	0.03
99	Nonclassifiable Establishments	3,021	2.85	0.03
45	Transportation by Air	2,156	2.80	0.03
13	Oil and Gas Extraction	31,535	2.72	0.03
80	Health Services	2,567	2.62	0.03
20	Food and Kindred Products	17,969	2.42	0.03
28	Chemicals and Allied Products	297,321	2.37	0.03
23	Apparel and Other Textile Products	1,010	2.35	0.03
21	Tobacco Products	3,269	2.27	0.02
1	Agricultural Production - Crops	7,424	1.27	0.01

Table IA.16: Summary Statistics, $\text{General}_{i,t}$ for Firms. This table shows percentile distributions for the firm measure of $\text{General}_{i,t}$, defined as the average score of patents at year t. The two columns Rolling (3) and Rolling (5) are rolling averages over the preceding years, with allotment for 1 missing year observation. The final column # Per Year is the number of patents a firm files in a given year. Scores are out of 100.

Percentile	Outside Sim	Rolling (3)	Rolling (5)	# Per Year
5th	0.58	0.74	0.8	1
25th	1.95	2.14	2.19	2
50th	3.37	3.47	3.5	5
75th	5.1	5.06	5.07	20
95th	8.81	8.32	8.13	212
Average	3.86	3.86	3.86	52.52
Standard Deviation	2.73	2.45	2.36	238.39
Firm-Year Observations	38,867	37,427	35,650	